CHANNEL ROUTING USING DISCRETE HAYAMI CONVOLUTION METHOD
WITH APPLICATIONS TO THE WATER EROSION PREDICTION
PROJECT (WEPP) MODEL

BY
LI WANG

A dissertation submitted in partial fulfillment of
the requirements for the degree of
DOCTOR OF PHILOSOPHY

WASHINGTON STATE UNIVERSITY
Department of Biological Systems Engineering
August 2012
To the Faculty of Washington State University:

The members of the Committee appointed to examine the dissertation of LI WANG find it satisfactory and recommend that it be accepted.

___________________________________
Joan Q. Wu, Ph.D., Chair

___________________________________
William J. Elliot, Ph.D.

___________________________________
Fritz R. Fiedler, Ph.D.

___________________________________
Sergey Lapin, Ph.D.
ACKNOWLEDGEMENTS

This research was supported in part through a USDA CSREES CEAP Grant (No. 2008-48686-04903) and a Graduate Research Assistantship at Washington State University. Supports from the College of Agricultural, Human, and Natural Resource Sciences, the Department of Biological Systems Engineering, and WSU Puyallup Research and Extension Center are greatly appreciated.

I thank my major advisor, Dr. Joan Q. Wu, for her consistent guidance, support, encouragement, and patience. She helped to secure the funding support to my research, and provided ample flexibility in my conceiving and conducting of the research, both much needed for fine-quality scholastic work. Most importantly, she devoted enormous time and effort helping me with manuscript writing. Without her, the completion of this study would have not been possible. I thank my committee members, Drs. William J. Elliot, Fritz R. Fiedler, and Sergey Lapin for their insightful guidance during this research, and their valuable suggestions and comments in developing the manuscripts. Dr. Elliot was instrumental to my learning of the WEPP model and its applications to real-world hydrological problems. Dr. Fiedler taught me computational hydrology and was helpful in site selection for model testing. Dr. Lapin provided generous help in numerical modeling. I am grateful to Dr. Dennis C. Flanagan and Mr. James R. Frankenberger for helping me to incorporate the channel routing routines developed in this doctoral research into the WEPP model. I thank Drs. Linda Hardesty and Hakjun Rhee for their helpful discussions, and those in our research group, Chaojun, Xiangdong, Dongjie, Liming, Mariana, Anurag, Emily, Hussin, Joe, and Natalie, for their support and friendship.

I especially thank my parents for their understanding and support. I thank my wife Shuhui, my daughter Kunxuan and son David, for their selfless support, understanding, and patience.
Chair: Joan Q. Wu

The primary goal of this research was to evaluate different channel routing methods that are suitable for watershed modeling and to develop channel routing routines that can be incorporated into typical watershed models. The specific objectives were (1) to investigate common channel-routing methods and apply them to a selected watershed model, i.e., the Water Erosion Prediction Project (WEPP) model; (2) to investigate the discrete Hayami convolution solution for linear diffusion-wave channel routing with uniformly-distributed lateral inflow; (3) to analyze the accuracy of the Muskingum-Cunge method for constant-parameter diffusion-wave channel routing with spatially and temporally variable lateral inflow; and, (4) to evaluate current channel-routing methods in the WEPP model.

For the discrete Hayami convolution channel routing, there were two ways of calculating the discrete kernel function values: using the exact point values or the center-averaged values. When the exact point values were used, the mass balance error of channel routing was dependent on the number $N$ of time steps on the rising limb of the kernel function. For the case applications, the
mass balance error was negligible when $N>1.8$. When the average kernel function values were used, however, the mass balance was always preserved since the integration of the discrete kernel function was always unity.

The constant-parameter Muskingum-Cunge (CPMC) method is generally second-order accurate. With specific discretizations such that the temporal and spatial intervals maintain a certain relationship, the CPMC can be third-order accurate. For channel routing in watershed modeling, lateral inflow is more important than upstream inflow and its accuracy can substantially affect the overall accuracy of channel routing. I derived the average lateral inflow terms in the second- and third-order accuracy CPMC. The derived equations indicate that for spatially and temporally variable lateral inflow, the average lateral inflow terms are affected not only by their spatial and temporal variations, but also by the numerical discretizations as well as wave celerity and diffusion coefficient of the channel flow.

Three channel routing methods, including the linear kinematic-wave, CPMC, and modified three-point variable-parameter Muskingum-Cunge, have been incorporated into WEPP. A sensitivity analysis for a hypothetical watershed showed that the simulated peak discharge was sensitive to time-step size, channel length, and channel bed slope, and not sensitive to channel width and channel bank inverse slope. A brief guide for selecting the appropriate channel routing method in WEPP applications under different watershed and channel conditions was provided.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACKNOWLEDGEMENTS</td>
<td>iii</td>
</tr>
<tr>
<td>ABSTRACT</td>
<td>iv</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>x</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>xi</td>
</tr>
<tr>
<td>1. GENERAL INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>1.1 Background</td>
<td>1</td>
</tr>
<tr>
<td>1.2 Objectives</td>
<td>3</td>
</tr>
<tr>
<td>1.3 Thesis outline</td>
<td>3</td>
</tr>
<tr>
<td>2. IMPLEMENTATION OF CHANNEL-ROUTING ROUTINES IN THE WATER EROSION PREDICTION PROJECT (WEPP) MODEL</td>
<td>5</td>
</tr>
<tr>
<td>2.0 Abstract</td>
<td>5</td>
</tr>
<tr>
<td>2.1 Introduction</td>
<td>5</td>
</tr>
<tr>
<td>2.2 Methods</td>
<td>8</td>
</tr>
<tr>
<td>2.2.1 Kinematic Wave Method</td>
<td>8</td>
</tr>
<tr>
<td>2.2.2 Muskingum-Cunge Method</td>
<td>13</td>
</tr>
<tr>
<td>2.2.3 Inflow hydrograph</td>
<td>15</td>
</tr>
<tr>
<td>2.2.4 Case simulations</td>
<td>16</td>
</tr>
<tr>
<td>2.3 Results</td>
<td>17</td>
</tr>
<tr>
<td>2.3.1 Verification of kinematic wave and Muskingum-Cunge solution</td>
<td>17</td>
</tr>
<tr>
<td>2.3.2 Effects of $\Delta x$ and $\Delta t$ on numerical solutions</td>
<td>17</td>
</tr>
<tr>
<td>2.3.3 Application to North Fork of Caspar Creek Watershed, CA</td>
<td>17</td>
</tr>
</tbody>
</table>
3. LINEAR DIFFUSION-WAVE CHANNEL ROUTING USING A DISCRETE HAYAMI CONVOLUTION METHOD

3.0 Abstract ............................................................................................................................... 28

3.1 Introduction ......................................................................................................................... 29

3.2 Diffusion wave model with lateral inflow .......................................................................... 32

3.2.1 General solution ........................................................................................................... 32

3.2.2 Discrete convolution solution with exact point kernel function values ...................... 33

3.2.3 Discrete convolution solution with average kernel function values ............................ 37

3.2.4 Integration error of the discrete kernel function ........................................................... 39

3.2.5 Channel water storage ................................................................................................. 39

3.3 A numerical experiment ...................................................................................................... 40

3.3.1 Comparison of discrete convolution solutions with Muskingum-Cunge solution ...... 40

3.3.2 Discrete convolution solution with point kernel function values ................................. 40

3.3.3 Discrete convolution solution with average kernel function values ............................ 41

3.3.4 Comparison of different calculation methods for discrete kernel function ........................ 41

3.3.5 Kernel function and temporal resolution ...................................................................... 41

3.3.6 Computing cost ............................................................................................................ 42

3.4 Applications ........................................................................................................................ 42

3.4.1 Asotin Creek, WA ........................................................................................................ 42

3.4.2 Clearwater River, ID .................................................................................................... 44

3.5 Summary and Conclusions ................................................................................................. 47
3.6 Tables and Figures .............................................................................................................. 49

4. ACCURACY OF MUSKINGUM-CUNGE METHOD FOR CONSTANT-PARAMETER
DIFFUSION-WAVE CHANNEL ROUTING WITH LATERAL INFLOW 71

4.0 Abstract ............................................................................................................................... 71

4.1 Introduction ......................................................................................................................... 72

4.2 Methods .............................................................................................................................. 74

4.2.1 Third-order accuracy CPMC method ........................................................................... 75

4.2.2 A numerical experiment ............................................................................................... 77

4.3 Results ................................................................................................................................. 80

4.4 Conclusions ......................................................................................................................... 81

4.5 Tables and Figures .............................................................................................................. 83

5. SENSITIVITY ANALYSIS OF CHANNEL-ROUTING PARAMETERS IN CURRENT
CHANNEL ROUTING METHODS IN THE WEPP MODEL 90

5.1 Time-step size ..................................................................................................................... 91

5.2 Channel bed slope .............................................................................................................. 92

5.3 Channel length .................................................................................................................... 92

5.4 Channel width ..................................................................................................................... 92

5.5 Channel inverse slope ........................................................................................................ 92

5.6 Summary ............................................................................................................................. 93

5.7 Tables and Figures .............................................................................................................. 94

6. SUMMARY AND CONCLUSIONS ......................................................................................... 105

REFERENCES ............................................................................................................................. 108
APPENDIX

A. Derivation of the third-order accuracy CPMC method for constant-parameter diffusion-wave channel routing with lateral inflow........................................................................... 115
B. Channel routing codes incorporated in WEPP................................................................. 122
  B.1. WSHCHR.FOR ........................................................................................................... 122
  B.2. CHRQIN.FOR .......................................................................................................... 134
  B.3. MANN.FOR ............................................................................................................. 138
  B.4. PMXCHR.INC ......................................................................................................... 141
  B.5. CCHRT.INC ........................................................................................................... 142
# LIST OF TABLES

3.1 USGS gauging stations on Clearwater River near Peck and Spalding, ID………….. 50

4.1 Accuracy of the second-order CPMC channel routing with lateral inflow for different time-step sizes ………………………………………………………………………. 84

4.2 Accuracy of the third-order CPMC channel routing with lateral inflow for different time-step sizes ………………………………………………………………………. 85

4.3 Accuracy of the CPMC channel routing with simplified calculation of lateral inflow (assuming uniformly distributed) for different time-step sizes …………………... 86

5.1 Input file *chan.inp* for channel routing in WEPP (later than v2010.1)………………. 95

5.2 An example of channel input file (*chan.inp*) ………………………………………... 96

5.3 Sensitivity of simulated peak discharge to time-step size …………………………… 97

5.4 Sensitivity of simulated peak discharge to channel bed slope …………………….. 98

5.5 Sensitivity of simulated peak discharge to channel length…………………………. 99

5.6 Sensitivity of simulated peak discharge to channel width…………………………... 100

5.7 Sensitivity of simulated peak discharge to channel inverse slope………………….. 101

5.8 A guide on selecting channel routing methods………………………………… 102
LIST OF FIGURES

2.1 An example of the double-exponential function $q_f(t)$ defined using the parameters from the WEPP daily hillslope surface runoff outputs, North Fork subwatershed of Caspar Creek Watershed, January 14, 1995. $t_c$, time of concentration, $t_d$, runoff duration, $q_p$, peak runoff rate, and $V$, runoff volume ........................................ 21

2.2 A hypothetical channel network, including four rectangular open channels, for the verification of the channel-routing module. The triangular inflow hydrographs for channels 1–3 were identical. The outflow hydrograph for channel 4 was calculated analytically and numerically as shown in Fig. 2.3 ........................................ 22

2.3 Comparison of analytical and numerical kinematic wave solutions and Muskingum-Cunge method for water flow out of channel 4 of the hypothetical channel network (Fig. 2.2), using a spatial interval of 300 m and a time step of 10 s ............ 23

2.4 Simulated hydrograph by the numerical kinematic wave solution using a time step of 10 min (KW, 10 min), and discharge differences by using the numerical kinematic wave solution with a 60-min time step (Diff., KW, 60 min), and the Muskingum-Cunge method with a 10-min or 60-min time step (Diff., MC, 10 and 60 min, respectively), for the North Fork of Caspar Creek, (a) January 1995, (b) a blowup of January 11–14, 1995 ................................................................. 24

2.5 Simulated peak flows for the North Fork of Caspar Creek, January 1995. KW, kinematic wave, MC, Muskingum-Cunge, CREAMS, Chemicals, Runoff, and Erosion from Agricultural Management Systems ........................................ 25
2.6 Comparison of simulated and observed hydrographs for the North Fork of Caspar Creek, January 1995

2.7 Daily water balance of the North Fork of Caspar Creek, January 1995, from the WEPP channel routing

3.1 Comparison of discrete convolution method and the constant-parameter Muskingum-Cunge (CPMC) method for a time step of 600 s. Note that simulated outflows from the discrete Hayami solution and the CPMC method largely overlap, and therefore only the differences between the two methods are shown

3.2 Discrete Hayami convolution solutions using point kernel function values for different sizes of time step. The solutions for 600-s and 60-s time steps largely overlap. For the time step of 3600 s, the calculated peak discharge is 1.7% lower than the theoretical value

3.3 Discrete Hayami kernel functions with point values for different sizes of time step

3.4 Discrete Hayami convolution solution using average kernel function values. The calculated peak discharge is not affected by the size of time step. The solutions for the 60-s and 600-s time steps largely overlap, and the calculated peak discharge is the same for all three time steps

3.5 Discrete Hayami kernel function from using average values for different sizes of time step
3.6 Comparison of discrete Hayami kernel functions using point- or average values (a), and the effect of the size of time step (b) and the number of time steps (c) on the integration error……………………………………………………………………………… 56

3.7 (a) Location of Asotin Creek and Clearwater River basins, northwestern US, (b) Asotin Creek basin, and (c) Lower Clearwater River basin…………………………………… 57

3.8 (a) Observed and simulated hydrographs for Asotin Creek, WA, in response to the rainfall and snowmelt events during December 1–16, 2007, and (b) observed temperature and precipitation for the same period…………………………………… 58

3.9 Comparison of discrete Hayami kernel functions using point- or average values (a), and the effect of the size of time step (b) and the number of time steps (c) on the integration error in the discrete Hayami kernel function for channel flow routing, Asotin Creek, WA, December 1–16, 2007………………………………… 59

3.10 Differences in simulated discharges between the Hayami discrete convolution methods with point- or average kernel function values and the constant-parameter Muskingum-Cunge (CPMC) method, Asotin Creek, WA, December 1–16, 2007… 60

3.11 Simulated channel water storage by different methods, Asotin Creek, WA, December 1–16, 2007. Note that the simulated storages using the discrete Hayami solution with average kernel function values and using the constant-parameter Muskingum-Cunge method largely overlap, thus only the former is shown………. 61

3.12 Observed hydrographs at the three gauging stations on Clearwater River and its tributary Potlatch River. Note different discharge scales used………………………… 62
3.13 Comparison of discrete Hayami kernel functions using point- or average values (a), and the effect of the size of time step (b) and the number of time steps (c) on the integration error in the discrete Hayami kernel function for channel flow routing, Potlatch River, ID, February 8–10, 2009…………………………………. 63

3.14 Observed inflow and simulated outflow for the Potlatch River (with the assumption of no lateral inflow), February 8–10, 2009. The results from the discrete Hayami solution using the average kernel function values and the CPMC method largely overlap…………………………………………………………….. 64

3.15 Differences in discharges between the discrete Hayami convolution solution and the CPMC method, Potlatch River, ID, February 8–10, 2009…………………………………. 65

3.16 Simulated channel storage using different methods, Potlatch River, ID, February 8–10, 2009. Note that the simulated storages from the discrete Hayami solution using the average kernel function values and the CPMC method largely overlap… 66

3.17 Comparison of discrete Hayami kernel functions with point- or average values (a), and the effect of the size of time step (b) and the number of time steps (c) on the integration error in the discrete Hayami kernel function for channel flow routing, upper reach, Clearwater River, ID, February 8–10, 2009…………………………………. 67

3.18 Simulated outflow and channel storage using different methods for the upper reach of the Clearwater River, ID, February 8–10, 2009. Note that the simulated outflow (or storage) from the discrete Hayami solution using the point- or average kernel function values largely overlap…………………………………………….. 68
3.19 Comparison of discrete Hayami kernel functions using point- or average values (a), and the effect of time step (b) and number of time steps (c) on integration errors in the discrete Hayami kernel function for channel routing, lower reach, Clearwater River, ID, February 8–10, 2009......................................................... 69

3.20 Simulated outflow and channel storage using different methods, lower reach, Clearwater River, ID, February 8–10, 2009.......................................................... 70

4.1 Schematic of the relationship between hillslopes and a channel segment in the Water Erosion Prediction Project (WEPP) model ............................................ 87

4.2 Root-mean-square errors (RMSE) of the simplified, second-, and third-order CPMC ................................................................................................................. 88

4.3 Analytical solution and differences in discharge between numerical and analytical solutions. (a) second-order accuracy CPMC, (b) third-order accuracy CPMC, and (c) simplified method. Note different scales used for difference in discharge in (a), (b), and (c) ........................................................................................................... 89

5.1 Effects of time-step size on input data selection for channel routing. (a) upstream channel inflow, (b) lateral channel inflow. Note the results for 60 s and 300 s largely overlap.......................................................... 103

5.2 Effects of channel length on simulated discharge. (a) Linear kinematic-wave (LKW), (b) constant-parameter Muskingum-Cunge (CPMC), and (c) modified three-point variable-parameter Muskingum-Cunge (MVPMC3).................. 104
CHAPTER 1
GENERAL INTRODUCTION

1.1 Background

The Water Erosion Prediction Project (WEPP) model is a process-based, continuous-simulation, distributed-parameter watershed hydrology and erosion prediction model (Flanagan and Livingston, 1995). It has been applied to agricultural and forested lands to simulate runoff and sediment yield from small watersheds and has been shown to be adequate in simulating daily water balance, including evapotranspiration (ET), soil water content, and streamflow (Conroy et al., 2006; Dun et al., 2009; Zhang et al., 2009; Williams et al., 2010). In the original versions of WEPP, channel flow is estimated using two options: the modified rational method used in the Erosion Productivity Impact Calculator (EPIC) model (Williams, 1990), and the regression equation used in the Chemicals, Runoff, and Erosion from Agricultural Management Systems (CREAMS) model (Knisel, 1980). These two methods, however, only give time to peak and peak runoff. Preliminary assessment in this study showed that these two methods tend to lead to over-estimated peak runoff. Additionally, it was assumed in these two methods that all runoff generated inside a watershed would leave the watershed outlet within a single day. This assumption may be valid for small watersheds (<260 ha) (Flanagan and Livingston, 1995), but can be erroneous for large watersheds.

Hydraulic or distributed channel-routing methods are based on mass and momentum conservation (Chow et al., 1988). Implementation of these methods into WEPP will significantly improve its applicability to large watersheds. Hydraulic channel-routing methods include the kinematic-wave, diffusion-wave, and dynamic-wave methods. Among the three, the kinematic-
wave method, by neglecting local and convective acceleration terms and the pressure force term (Chow et al., 1988), is the simplest form of the St. Venant equations and has been widely used (Singh, 2001). The diffusion-wave method is also widely used because it is easier to implement compared to the dynamic-wave method, yet still gives sufficiently accurate results (Moussa, 1996; Moussa and Bocquillon, 1996; Singh et al., 1997; Wang et al., 2003; Fan and Li, 2006; Moramarco et al., 2008).

The diffusion-wave equations can be simplified to one single equation given a relationship between the discharge and its cross-sectional area, such as the Manning’s or Chezy’s equation (Moussa, 1996; Fan and Li, 2006). With the assumption of constant wave celerity and diffusion coefficient, this equation can be solved analytically to give the convolution solution (Hayami, 1951; Ogata and Banks, 1961; Moussa, 1996; Fan and Li, 2006). The convolution is usually solved by numerical integration. For a watershed with hundreds to thousands of channels, the numerical integration may not be practical because it is highly time-consuming.

The Muskingum-Cunge method numerically solves the kinematic-wave equation considering both storage and transport. By matching the numerical and the physical diffusion, it can be used to simulate diffusion wave (Cunge, 1969; Ponce, 1995; Ponce et al., 1996). Bajracharya and Barry (1997) and Szel and Gaspar (2000) examined the Muskingum-Cunge channel routing without lateral inflow, and derived relationships between the spatial and temporal intervals. With these relationships, the Muskingum-Cunge method can be second- or third-order accurate. In a watershed simulation, lateral inflow is often more important than upstream inflow in modeling channel flows. Hence, the accuracy of the lateral inflow term in the Muskingum-Cunge equation becomes a major factor for accuracy of Muskingum-Cunge channel routing.
1.2 Objectives

The main goal of this study was to investigate different channel routing methods that are suitable for watershed modeling and to develop channel routing routines that can be incorporated into typical watershed models, such as WEPP. The specific objectives of this dissertation were:

1. To investigate different channel routing methods and apply them to the WEPP model. These methods include the linear kinematic-wave (LKW), constant-parameter Muskingum-Cunge (CPMC), and modified three-point variable-parameter Muskingum-Cunge (MVPMC3);

2. To investigate the discrete Hayami convolution solution for linear diffusion-wave channel routing with uniformly-distributed lateral inflow and its applications and limitations;

3. To analyze the accuracy of the Muskingum-Cunge method for constant-parameter diffusion-wave channel routing with spatially and temporally variable lateral inflow; and

4. To evaluate current channel routing methods in the WEPP model.

1.3 Thesis outline

This dissertation includes a general introduction, three chapters in the form of technical manuscripts of which one has been published and the other two submitted for publication, a chapter evaluating the channel routing methods in the current WEPP, and a chapter presenting major conclusions. Chapter 1 provides a brief introduction to the dissertation, background, and objectives. Chapter 2 presents the implementation of the kinematic-wave and Muskingum-Cunge channel routing methods in the WEPP model. Results using the two new channel routing methods and CREAMS were compared in a case application. In Chapter 3, a discrete Hayami convolution solution for diffusion-wave channel routing with lateral inflow is investigated. Two methods of calculating the Hayami kernel function values and their effect on mass-balance error in channel routing are analyzed. Chapter 4 presents the second- and third-order accuracy
constant-parameter Muskingum-Cunge methods for channel routing with temporally and spatially variable lateral inflow. The second- and third-order accuracy average lateral inflow terms in the Muskingum-Cunge equation are derived. Chapter 5 summarizes current channel routing methods in the WEPP model, with a focus on sensitivity analysis and general guidelines for using individual channel routing methods. User settings for running WEPP with channel routing and output generation are described, and peak runoff calculated by the new channel routing methods and the original methods are compared. Chapter 6 presents the important conclusions of the dissertation. Tables and figures are presented at the end of each chapter, following the format for manuscript submission to the specific refereed technical journal. References and appendices are listed at the end of the dissertation.
CHAPTER 2
IMPLEMENTATION OF CHANNEL-ROUTING ROUTINES IN THE WATER EROSION PREDICTION PROJECT (WEPP) MODEL

2.0 Abstract

The Water Erosion Prediction Project (WEPP) model is a process-based, continuous-simulation, watershed hydrology and erosion model. It is an important tool for water erosion simulation owing to its unique functionality in representing diverse landuse and management conditions. Its applicability is limited to relatively small watersheds since its current version does not simulate flow in permanent channels. In this study we developed a channel-routing module to simulate water flow in a permanent channel network. The module can utilize two methods: numerical kinematic-wave method and Muskingum-Cunge method. Results showed that, for appropriate temporal and spatial discretizations, both numerical solutions compared well with analytical solution of kinematic wave equations for simplified cases; otherwise, numerical dissipation from the kinematic wave solution, and numerical dispersion from the Muskingum-Cunge solution would occur.

2.1 Introduction

Water resources are critical to a nation’s security (Lowdermilk, 1953). The quantity and quality of surface water resources depends on watershed attributes such as climate, geology, and topography, and watershed conditions affected by vegetation and human and natural disturbances (Fangmeier et al., 2006). Watersheds are frequently managed to ensure that runoff from the watershed meets conditions suitable for downstream beneficial uses. One of the techniques used

---

for watershed management is to model the effect of human disturbances on water quantity and quality, and to compare the results associated with different management practices. A “model” is a mathematical or qualitative representation of nature. It involves an understanding of the analysis area, including the identification of the important features and processes, such as topography, soil properties, vegetation, and climate as well as their interactions. Models provide answers to a variety of management questions, e.g., “what watershed changes are anticipated as a result of proposed fuel management activities?” (Elliot et al., 2007).

A number of streamflow models have been developed (Borah and Bera, 2003). Some perform better in predicting flows for larger channels over longer time steps (e.g., annual, long-term average), while others may be more suitable for smaller spatial scale and shorter time steps (e.g., seconds or minutes) (Borah and Bera, 2004).

The Water Erosion Prediction Project (WEPP) model is a process-based, continuous-simulation, watershed hydrology and erosion prediction model. The model inputs include climate, structure of hillslopes and channels, physical properties of hillslopes and channels (topography, geometry, geology), and, landuse, vegetation, and management practices. WEPP has been applied to agricultural and forested lands to simulate runoff and sediment yield from small watersheds. The model has been shown to be adequate in simulating daily water balance, including soil water content and streamflow (Dun et al., 2009; Zhang et al., 2009; Williams et al., 2010).

Presently, the applicability of WEPP is limited to relatively small watersheds (<260 ha) since it does not simulate flow in permanent channels (Flanagan and Livingston, 1995). In WEPP, as summarized in (Ascough et al., 1995), channel runoff is estimated using two options: the modified rational method used in the Erosion Productivity Impact Calculator (EPIC) model.
(Williams, 1990), and the regression equation used in the Chemicals, Runoff, and Erosion from Agricultural Management Systems (CREAMS) model (Knisel, 1980). Preliminary studies showed that WEPP could properly simulate daily water balance, but tended to over-predict peak flow.

In a sediment erosion study in the North Caspar Creek Watershed, CA, a long-term US Forest Service experimental watershed, Conroy et al. (2006) coupled WEPP with a hydrodynamic sediment transport model, CCHE1D (Wu and Vieira, 2002). The hillslope outputs from WEPP simulation were processed with an interface program and were then used as inputs to CCHE1D. Bdour and Papanicolaou (2008) studied sediment erosion and transport in the Red River watershed, ID, by coupling WEPP with two hydrodynamic sediment transport models based on the hypsometric curve approach and the virtual velocity concept, respectively. The sediment from subwatersheds simulated by WEPP was routed to the watershed outlet using the two sediment transport models. The additional data processing in coupling WEPP with streamflow and sediment transport models can be difficult for general users, which greatly limits the application of WEPP to larger watersheds.

Most current physically-based flood routing models are based on the St. Venant continuity and momentum equations (Chow et al., 1988). The momentum equation includes force terms to describe water flow within a channel: local and convective acceleration forces, pressure force, gravitational force, and frictional force. Three types of hydraulic methods, which solve the St. Venant equations, have been used for streamflow routing. They are the dynamic wave, diffusion wave, and kinematic wave methods. The dynamic wave method includes all the momentum terms in the St. Venant equations. It is suitable for streams with gentle channel slopes and downstream controls (natural or man-made) where inertial and pressure forces and backwater
effects are considered. The dynamic wave method is often numerically unstable and expensive to compute. The diffusion wave method neglects the local and convective acceleration terms in the St. Venant equations. The kinematic wave method is the simplest form of the St. Venant equations and has been widely used (Singh, 2001). It neglects local and convective acceleration terms as well as the pressure force term, and only considers the gravitational and frictional force terms. The Muskingum-Cunge method numerically solves the kinematic wave equation, and considers both a storage and a transport concept. It may be considered as a hybrid approach (Ponce, 1995).

In this study, a channel-routing module was developed and incorporated into the WEPP model (v2008.9). A numerical solution of the kinematic wave was implemented and verified against the analytical solution for a simplified case. Additionally, the Muskingum-Cunge method was implemented and its solution was compared with the analytical and numerical kinematic wave solutions. The modified WEPP model was applied to simulate watershed runoff from the North Fork of Caspar Creek Watershed, CA. In addition to computing daily water balance, the new channel routing module will route inflow from upstream and lateral flow through the stream banks to channel outlets at each specified time step.

2.2 Methods

2.2.1 Kinematic Wave Method

Water flow in a one-dimensional channel can be described with the St. Venant equations (Chow et al., 1988), including a continuity equation and a momentum equation, as

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = q, \quad (2.1)$$

and
\[
\frac{1}{A} \frac{\partial Q}{\partial t} + \frac{1}{A} \frac{\partial}{\partial x} \left( \frac{Q^2}{A} \right) + g \frac{\partial y}{\partial x} - g (S_o - S_f) = 0
\]  

(2.2)

where \( A = A(x, t) \) and \( Q = Q(x, t) \) are the cross-sectional area and discharge of the channel flow, respectively, \( q = q(t) \) is lateral inflow, \( y = y(x, t) \) is depth of water, \( g \) is gravitational acceleration, \( S_o \) is the channel bed slope, \( S_f \) is the slope of the energy-grade line or friction slope, \( x \) is distance, and \( t \) is time.

For watershed modeling, the demand in data and computation is high due to the complexity of the highly dynamic and interactive hydraulic and hydrological processes at large scales. For water flow in a watershed where the slopes of stream channels tend to be steep and the gravitational force is dominant, the first three terms in equation (2.2) are negligible, and the gravity and friction forces are assumed to be balanced, leading to

\[
S_o - S_f = 0.
\]

(2.3)

The approach characterizing water flow by equations (2.1) and (2.3) is called the kinematic wave method. From equation (2.3), \( S_f = S_o \). For uniform, turbulent channel flow, Manning’s equation can be used to describe relationship between discharge and channel geometry (Chow et al., 1988):

\[
Q = \frac{S_o^{\frac{1}{2}}}{n} AR^{\frac{2}{3}}
\]

(2.4)

where \( n \) is the Manning’s roughness coefficient, and \( R \) is the hydraulic radius (ratio of the cross-sectional area to the wetted perimeter).

From equation (2.4), we have

\[
\frac{\partial Q}{\partial t} = \frac{dQ}{dA} \frac{\partial A}{\partial t} = C_s \frac{\partial A}{\partial t}
\]

(2.5)
where $C_k = \frac{dQ}{dA}$ is the kinematic wave celerity. We present the calculation of $C_k$ in the following for two different cross-sectional configurations of channels.

### 2.2.1.1 Depth of water and kinematic wave celerity

For a rectangular channel,

$$A = By, \quad R = \frac{By}{B + 2y}$$

where $B$ is the channel bottom width, and $y$ is the depth of water. We can then rewrite equation (2.4) as

$$Q = \frac{S_y^{\frac{3}{2}}}{n} By \left(\frac{By}{B + 2y}\right)^{\frac{2}{3}}. \quad (2.6)$$

For a known discharge $Q_0$, $y$ can be calculated using Newton’s method (Chow et al., 1988).

The kinematic wave celerity $C_k$ for a rectangular channel can be calculated from

$$C_k = \left(1 + \frac{2B}{3(B + 2y)}\right) \frac{Q}{By} \quad (2.7)$$

For a wide rectangular channel, $B >> y$, $C_k$ can be simplified as (Chow et al., 1988; Wong, 2003)

$$C_k = \frac{5Q}{3By}$$

For a triangular channel,

$$A = sy^2, \quad R = \frac{sy}{2\sqrt{1 + s^2}}$$

where $s$ is the inverse slope of the channel bank. We have
\[ y = \left( \frac{A}{s} \right)^{\frac{1}{2}}, \quad R = \frac{s \left( \frac{A}{s} \right)^{\frac{1}{2}}}{2\sqrt{1 + s^2}}, \]

and (as in Ponce (1995))

\[ Q = \frac{S_o^{\frac{1}{2}}}{n} \cdot \frac{(sA)^{\frac{1}{3}}}{2^{\frac{2}{3}} (1 + s)^{\frac{1}{3}}} = \frac{S_o^{\frac{1}{2}}}{2^{\frac{2}{3}} n} \left( \frac{s}{1 + s^2} \right)^{\frac{1}{3}} A^{\frac{4}{5}}. \quad (2.8) \]

For a triangular channel and a known \( Q_0 \), \( y \) can be directly calculated from the following equation

\[ y = 2^{\frac{1}{6}} \left( 1 + s^2 \right)^{\frac{1}{6}} \left( nQ_0 \right)^{\frac{3}{8}} \frac{S_o^{\frac{1}{6}} S^\frac{5}{8}}{S_o^{\frac{7}{6}}}. \quad (2.9) \]

The kinematic wave celerity for a triangular channel can be derived from equation (2.8) as Ponce (1995), Tewold and Smithers (2006)

\[ C_k = \frac{4Q}{3y^2}. \quad (2.10) \]

2.2.1.2 Analytical method

Combined with Manning’s equation (2.4), equation (2.1) can be solved analytically. For a given discharge \( Q \) at the channel inlet, its kinematic wave celerity can be calculated using equations (2.7) and (2.10) for a rectangular and a triangular channel, respectively. Its travel time \( t \) to the outlet is (Chow et al., 1988)

\[ t = \frac{L}{C_k} \quad (2.11) \]

where \( L \) is the channel length.
2.2.1.3 Finite difference method

With

\[
\frac{\partial Q}{\partial t} = \frac{dQ}{dA} \frac{\partial A}{\partial t} = C_k \frac{\partial A}{\partial t},
\]

eliminating \( A \) from equation (2.1), we obtain

\[
\frac{\partial Q}{\partial t} + \frac{1}{C_k} \frac{\partial Q}{\partial t} = q. \tag{2.12}
\]

The kinematic wave equations were solved by a finite difference method. Using a linear implicit scheme (Chow et al., 1988), we can express the partial differential equation (2.12) as

\[
\frac{Q_{i+1}^{j+1} - Q_{i}^{j+1}}{\Delta x} + \frac{1}{C_k} \frac{Q_{i+1}^{j+1} - Q_{i}^{j+1}}{\Delta t} = \bar{q} \tag{2.13}
\]

where \( i \) and \( j \) refer to the spatial and temporal nodes, respectively, \( Q_{i}^{j} = Q(i\Delta x, j\Delta t) \), \( \Delta x \) is the spatial increment, and \( \Delta t \) is the time step, and

\[
\bar{q} = \frac{q_{i+1}^{j+1} + q_{i+1}^{j+1}}{2}.
\]

In assessing and calibrating the channel-routing model, a user can define the time step \( \Delta t \) and space step \( \Delta x \) to test their effects on numerical dissipation and dispersion. In this study, we set \( \Delta t \) as one tenth of the weighted average \( t_c \) for all hillslopes to maintain appropriate temporal resolution, with defined upper and lower bounds. For the kinematic wave method, \( \Delta x \) for each channel is calculated based on the peak inflow and the Courant-Friedrichs-Lewy (CFL) condition to minimize the dissipation effect (Chow et al., 1988; Thomas, 1995). The Courant number was assumed equal to or slightly less than 1, or \( \Delta x \geq C_{k, peak} \Delta t \), where \( C_{k, peak} \) was calculated following equation (2.7) for a rectangular channel or (2.10) for a triangular channel using the peak inflow discharge.
$C_k$ in equation (2.13) can be estimated the same way as $C_k$, peak using the average discharge $\bar{Q}$:

$$\bar{Q} = \frac{Q^{j+1}_{i+1} + Q^{j+1}_i}{2}.$$  

Solving equation (2.13) for the unknown $Q^{j+1}_{i+1}$, we obtain

$$Q^{j+1}_{i+1} = \frac{\Delta t}{\Delta x} \left[ \frac{Q^j_i - C_k Q^j_{i+1}}{C_k + \Delta \bar{q}} \right].$$  \hspace{1cm} (2.14)

It should be noted that, when the inflow hydrograph contains wave shocks, the numerical kinematic wave method may lead to mass balance and travel speed errors (LeVeque, 1992); but under normal conditions, the mass balance errors may be neglected (Gasiorowski and R. Szymkiewicz, 2007). In our study, the daily inflow hydrograph from a hillslopes to a channel was simplified as a double-exponential function, the small mass-balance errors resulting from temporal and spatial discretization were further eliminated by assuming that the water storage in a channel cannot be negative, and the residual water that has not flowed out of the outlet of a channel remains as storage of the channel.

**2.2.2 Muskingum-Cunge Method**

The Muskingum method calculates channel outflow based on channel inflow and channel storage, with the assumption of a linear relationship of channel storage versus channel inflow and outflow (Chow et al., 1988; Ponce, 1995). The Muskingum-Cunge method combines the Muskingum method with additional physical information, such as channel geometry and channel flow, and matches the numerical diffusion with the hydraulic diffusion of the channel flow (Ponce, 1995). The unknown discharge $Q^{j+1}_{i+1}$ at $(x_{i+1}, t_{j+1})$ can be calculated as (Wang et al., 2006)
\[ Q_{i+1}^{j+1} = C_1 Q_i^{j+1} + C_2 Q_i^j + C_3 Q_{i+1}^j + C_4 \Delta x \bar{q} \]  \hspace{1cm} (2.15)

The coefficients \( C_1, C_2, C_3, \) and \( C_4 \) in (2.15) are defined as

\[ C_1 = \frac{\Delta t - 2KX}{2K(1 - X) + \Delta t}, \]

\[ C_2 = \frac{\Delta t + 2KX}{2K(1 - X) + \Delta t}, \]

\[ C_3 = \frac{2K(1 - X) - \Delta t}{2K(1 - X) + \Delta t}, \]

and

\[ C_4 = \frac{2\Delta t}{2K(1 - X) + \Delta t} \]

where

\[ K = \frac{\Delta x}{C_k}, \]

and

\[ X = \frac{1}{2} \left( 1 - \frac{Q_r}{BC_k S_o \Delta x} \right) \]

where \( B \) is the known channel width for a rectangular channel and \( B = 2sy \) for a triangular channel, \( Q_r \) is the reference discharge, and \( C_k \) is estimated using \( Q_r \) from equation (2.7) for a rectangular channel or (2.10) for a triangular channel. For a greater computational efficiency, \( \Delta x \) may be specified as the length of the simulated channel (Chow et al., 1988), and \( Q_r \) as the peak inflow discharge. It has been shown that the mass balance errors from the Muskingum-Cunge method are negligible under normal conditions (Ponce and Chaganti, 1994).
2.2.3 Inflow hydrograph

Within a stream network, the upstream end of a reach can receive inflow from one to three channels, or a single hillslope. Additional inflow can be from hillslopes on either side of the reach. Hillslope flux can include surface runoff and subsurface lateral flow. In the future, additional improvement may be made by modeling the ground-water base flow.

The daily hydrograph outputs from a WEPP hillslope simulation include the following parameters: hillslope runoff duration \((t_d)\), time of concentration \((t_c)\), peak runoff rate \((q_p)\), runoff volume \((V)\), and subsurface runoff volume \((V_{sb})\), with an assumed duration of 24 hours. These parameters define an input hydrograph of variable flow rate combining both hillslope surface runoff and subsurface lateral flow for each channel reach.

The current version of WEPP estimates the peak runoff rate for each channel reach as a fraction of peak precipitation intensity on the hillslopes (Elliot et al., 2007). Conroy et al. (2006) used a linear equation for the rising limb of the hydrograph, and regression equations derived from observed data for the falling limb, considering \(V\) and \(t_c\) from the WEPP hillslope simulation outputs. In the present study, we used a double-exponential function defined from all of the WEPP daily hydrograph parameters (Fig. 2.1) to describe a continuous hillslope surface flow hydrograph for channel routing. Such an approach was also used in WEPP for fitting rainfall intensity data (Nicks et al., 1995). The double-exponential function is given by

\[
q_f(t) = \begin{cases} 
q_pe^{bt-t_c} & 0 \leq t \leq t_c \\
q_pe^{-dt-t_c} & t_c < t \leq t_d 
\end{cases}
\]  

(2.16)

where \(q_f(t)\) is the calculated runoff rate, and \(t_c\) is assumed to be the time that the peak runoff rate occurs. The coefficients \(b\) and \(d\) can be obtained using Newton’s method with the assumption \[ q_f(0) = q_f(t_d) \]
and mass conservation

\[ \int_{0}^{t_f} q_f(t) \, dt = V. \]

### 2.2.4 Case simulations

For the purpose of model verification and assessment of model performance, we simulated water flow in a hypothetical channel network (Fig. 2.2) using the channel-routing module. The channel network consisted of four rectangular open channels with triangular inflow hydrograph for channels 1–3.

We then applied the modified WEPP model to the North Fork of Caspar Creek Watershed, CA. The Caspar Creek Experimental Watershed is located in the middle of Jackson Demonstration State Forest, south of Fort Bragg, CA. It has been studied for the effects of forest management on runoff and soil erosion since 1962 (Keppeler et al., 2003). The North Fork drains an area of 473 ha (1.83 mi²). Gauging stations have been installed for 21 sites to monitor streamflow (Keppeler et al., 2003).

Except for the study by Conroy et al. (2006) coupling WEPP with the CCHE1D model to evaluate upland erosion and channel sediment transport, systematic simulation for overland and channel flow for this area is lacking. As in Conroy et al. (2006), we discretized the North Fork sub-watershed of Caspar Creek Watershed into 88 hillslopes and 35 channels.

To compare the effects of time step and assess the differences in the analytical and numerical kinematic wave solutions as well as the Muskingum-Cunge method, we ran the WEPP model using time steps of 10 or 60 min in the two numerical methods. The simulation results were also compared with the observed hydrograph at the outlet of the North Fork sub-watershed of Caspar Creek watershed.
2.3 Results

2.3.1 Verification of kinematic wave and Muskingum-Cunge solution

With an appropriate temporal and spatial discretization, the results from the numerical kinematic wave solution and the Muskingum-Cunge method compared well with the analytical kinematic wave solution (Fig. 2.3).

2.3.2 Effects of $\Delta x$ and $\Delta t$ on numerical solutions

The numerical kinematic wave method led to dissipation, or wave attenuation with time (Thomas, 1995). The dissipation increased with the size of temporal and spatial increments in this study, as reported in previous studies (Chow et al., 1988).

The numerical dissipation from the Muskingum-Cunge method was negligible. But numerical dispersion, i.e., waves with different wave lengths travel at different speeds (Thomas, 1995), was observed if $\Delta t$ was too small for a selected $\Delta x$, or $\Delta x$ was too large for a selected $\Delta t$. This was because parameter $C_1$ in equation (2.15) would become negative, leading to decreased flow at the wave front (Fig. 2.3).

2.3.3 Application to North Fork of Caspar Creek Watershed, CA

The simulated hydrograph by using the numerical kinematic wave solution with a 10-min time step (KW, 10 min) is shown in Fig. 2.4. The results from the numerical kinematic wave solution with a 60-min time step (KW, 60 min), the Muskingum-Cunge method with a 10-min or 60-min time step (MC, 10 min and MC, 60 min, respectively) are shown as differences from this hydrograph (Fig. 2.4). The additional rises or dips (usually at the beginning or end of a day) caused by using the KW, 60 min were mainly because of resolution difference. The results from the Muskingum-Cunge method yielded a slightly higher peak flow than the numerical kinematic wave solution, and showed some numerical dispersion. The numerical dispersion from using
\( \Delta t = 60 \text{ min} \) was higher than using \( \Delta t = 10 \text{ min} \). The simulated daily peak flows by the kinematic wave and Muskingum-Cunge methods are shown in Fig. 2.5. Compared with the CREAMS method in the original WEPP that led to high flow peaks, the kinematic wave and Muskingum-Cunge methods had lower peak flows. The simulated hydrograph was also compared with observed streamflow data (Fig. 2.6) for January 1995. The first simulated peak is lower than the observed, but the others are agreeable with the observed. The total storage in all the channels upstream from the outlet of the North Fork Caspar Creek subwatershed is usually smaller than the amount of daily inflow or outflow in January (Fig. 2.7), because of the high velocity of water flow. During a dry season, surface-water storage in the sub-watershed may be larger than the amount of inflow or outflow (not shown). Fig. 2.7 also indicates that the time of the largest storage may not coincide with the time of the largest flow event.

### 2.4 Summary and Conclusions

A channel-routing module was developed to simulate water flow in a channel network using either the numerical kinematic wave method or Muskingum-Cunge method. With appropriate temporal and spatial discretization, both numerical solutions compared well with the analytical solution of kinematic wave equations for simplified cases. The kinematic wave method was more robust even for nonlinear waves, but the resultant numerical dissipation increased with the sizes of temporal and spatial discretization. The Muskingum-Cunge method was computationally more efficient, and the resultant peak attenuation was smaller. From this point of view, we suggest the Muskingum-Cunge method to be used for calculating peak flows in erosion estimation, as in the WEPP model. Future work includes code implementation to minimize numerical dispersions in the Muskingum-Cunge method, and testing the channel-routing routines
newly incorporated in the WEPP model with streamflow data from different geographic localities.
2.5 Figures
Figure 2.1. An example of the double-exponential function $q_f(t)$ defined using the parameters from the WEPP daily hillslope surface runoff outputs, North Fork subwatershed of Caspar Creek Watershed, January 14, 1995. $t_c$, time of concentration, $t_d$, runoff duration, $q_p$, peak runoff rate, and $V$, runoff volume.
Figure 2.2. A hypothetical channel network, including four rectangular open channels, for the verification of the channel-routing module. The triangular inflow hydrographs for channels 1–3 were identical. The outflow hydrograph for channel 4 was calculated analytically and numerically as shown in Fig. 2.3.
Figure 2.3. Comparison of analytical and numerical kinematic wave solutions and Muskingum-Cunge method for water flow out of channel 4 of the hypothetical channel network (Fig. 2.2), using a spatial interval of 300 m and a time step of 10 s.
Figure 2.4. Simulated hydrograph by the numerical kinematic wave solution using a time step of 10 min (KW, 10 min), and discharge differences by using the numerical kinematic wave solution with a 60-min time step (Diff., KW, 60 min), and the Muskingum-Cunge method with a 10-min or 60-min time step (Diff., MC, 10 and 60 min, respectively), for the North Fork of Caspar Creek, (a) January 1995, (b) a blowup of January 11–14, 1995.
Figure 2.5. Simulated peak flows for the North Fork of Caspar Creek, January 1995. KW, kinematic wave, MC, Muskingum-Cunge, CREAMS, Chemicals, Runoff, and Erosion from Agricultural Management Systems.
Figure 2.6. Comparison of simulated and observed hydrographs for the North Fork of Caspar Creek, January 1995.
Figure 2.7. Daily water balance of the North Fork of Caspar Creek, January 1995, from the WEPP channel routing.
CHAPTER 3
LINEAR DIFFUSION-WAVE CHANNEL ROUTING USING A DISCRETE HAYAMI
CONVOLUTION METHOD

3.0 Abstract

The convolution of an input with a response function has been widely used in hydrology as a
means to solve various problems analytically. Due to the high computation demand in solving
the functions using numerical integration, it is often advantageous to use the discrete convolution
instead of the integration of the continuous functions. This approach greatly reduces the amount
of the computational work; however, it increases the possibility for mass balance errors. In this
study, we analyzed the characteristics of the kernel function for the Hayami convolution solution
to the linear diffusion-wave channel routing with distributed lateral inflow. We propose two
ways of selection of the discrete kernel function values: using the exact point values or using the
center-averaged values. Through a hypothetical example and the applications to Asotin Creek,
WA and the Clearwater River, ID, we showed that when the point kernel function values were
used in the discrete Hayami convolution solution, the mass balance error of channel routing is
dependent on the number of time steps on the rising limb of the Hayami kernel function. The
mass balance error is negligible when there are more than 1.8 time steps on the rising limb of the
kernel function. The fewer time steps on the rising limb, the greater risk of high mass balance
errors. When the average kernel function values are used for the Hayami discrete convolution

---

1 A modified version of this chapter has been submitted for publication: Wang, L., Wu, J.Q., Elliot, W.J., Fiedler,
(in review)
solution, however, the mass balance is always maintained, since the integration of the discrete kernel function is always unity.

3.1 Introduction

In a stream network, the discharge from a given reach depends on the upstream inflow, the storage within the reach, and any gains from, or losses to, lateral flow. Storage is dependent on the micro topography of the valley along the reach. Lateral flow may originate from the interflow through shallow surface layers (Watson and Burnett, 1995), e.g., in steep forests (Dun et al., 2009), or through surface and groundwater interactions. Whether the lateral flow is positive or negative will depend on the relative hydraulic gradient between the channel stage and the adjacent water table.

Open-channel flow is typically simplified as one-dimensional flow processes described by the Saint-Venant equations, which consist of the continuity equation and the momentum equation (Chow et al., 1988; Singh et al., 1997). There are three methods based on the simplifications to the momentum equation: the dynamic-wave method that considers all the terms in the momentum equation, the diffusion-wave method that neglects the acceleration terms, and the kinematic-wave method that neglects both the acceleration and pressure force terms (Chow et al., 1988; Singh et al., 1997). Among these three methods, the diffusion-wave method is widely used because it is easier to implement compared to the dynamic-wave method, yet still gives sufficiently accurate results (Moussa, 1996; Moussa and Bocquillon, 1996; Singh et al., 1997; Wang et al., 2003; Fan and Li, 2006; Moramarco et al., 2008).

Combined with the Manning’s equation or the Chezy’s equation, the diffusion-wave equations can be simplified to one single equation (Moussa, 1996; Fan and Li, 2006). With the assumption of constant wave celerity and diffusion coefficient, this equation has been solved
analytically to give the convolution solution (Hayami, 1951; Ogata and Banks, 1961; Moussa, 1996; Fan and Li, 2006). For simplified diffusion-wave channel routing without lateral inflow, we can obtain the analytical convolution equation and solve it analytically or by numerical integration techniques. Assuming no lateral inflow, Hayami (1951) solved the linear diffusion-wave model using perturbation theory, and obtained the analytical solution in terms of water depth in convolution form. He also found that, for the linear diffusion wave, the unit-graph method could be used to approximate the analytical solution for appropriate time step sizes. Tingsanchali and Manandhar (1985) developed the convolution solution for diffusion-wave channel routing considering lateral inflow and backwater effect. Through a Laplace transform, Moussa (1996) obtained the convolution solution for the linear diffusion-wave model taking into consideration of distributed lateral inflow with a steady-state initial condition. Fan and Li (2006) further extended the solution for linear diffusion-wave channel routing with distributed or concentrated lateral inflow, and analyzed outflow change as affected by different boundary conditions.

The method of convolving an input with a kernel function has been applied to various arenas in hydrology to obtain the analytical solution of the output. Dooge (1959) convoluted rainfall data with the instantaneous unit hydrograph to calculate the rainfall response at an outlet of a channel or watershed. Barlow et al. (2000) and Hantush (2005) applied the convolution solution to one-dimensional surface water-groundwater interactions, considering the effects of channel inflow, lateral inflow, groundwater recharge, and hydraulic gradients between surface and groundwater. Olsthoorn (2008) presented case applications of the convolution method to analysis of fluctuations of groundwater level as affected by local and remote recharges, and drawdown calculation of pumping tests. McGuire et al. (2005) applied the convolution method to examining
tracer movement to estimate water residence times within a watershed. Ogata and Banks (1961) solved the diffusion-wave equation (in the same form as for channel flow) for mass transport in groundwater flow, and obtained the convolution solution.

The major advantages of the analytical solutions over numerical methods in channel-flow routing are that the former are usually unconditionally stable and computationally more efficient (Hayami, 1951; Chaudhry, 1993, p.370; Moussa, 1996; Fan and Li, 2006). However, analytical solutions can only be obtained for limited, special cases. For a simplified case with a constant inflow at the upper boundary and homogenous initial condition, Ogata and Banks (1961) found that the convolution solution can be expressed in terms of error functions, which have tabulated values and can be readily solved using computer programs. When the lateral inflow is a significant component of the channel inflow, the wave celerity and diffusion coefficient can no longer be considered constant for the entire channel reach. Moussa and Bocquillon (1996) suggested the use of numerical methods in such cases. In general, we need to solve the convolution equations by numerical integration, which is time-consuming and limits the applications of the analytical solution (Munier et al., 2008). To obtain the analytical solution efficiently when the inflows are at a constant temporal interval, we can apply discrete methods to approximate the continuous convolution solution (Chow et al., 1988, p. 204; Long, 2009).

In a discrete method, the input is given as a series of data values at a specified temporal interval, and the solution is obtained at the same temporal coordinates as the input. Discrete methods have been widely used in hydrology, such as in the unit hydrograph method (Dooge, 1959; Chow et al., 1988), the discrete cascade models based on the Muskingum reservoir method (O’Connor, 1976; Perumal, 1994; Camacho and Lees, 1999; Perumal et al., 2007), the discrete convolution methods for estimating surface water-groundwater interactions (Barlow et al., 2000;
Hantush et al., 2002; Hantush, 2005), and the discrete convolution method for diffusion-wave channel routing (Tingsanchali and Manandhar, 1985).

In all aforementioned discrete methods, the response function should sum to unity for mass balance. Chow et al. (1988, p. 218) suggested to scale up or down the corresponding response function values if their summation does not meet this criterion. This remedy is easy to implement but it may shift the kernel function curve. Hence, this approach is not recommended when the accuracy of peak time is important for a specific study. The cause of the mass-balance error is due to the coarse temporal resolution. In order to achieve more accurate results of the discrete convolution solution, Barlow et al. (2000) proposed to use smaller time steps. In most applications, the time step in the observation data is fixed, and it would be important to know whether the given temporal interval is sufficiently fine.

The objective of this study was to investigate the characteristics of the Hayami kernel function to (i) develop criteria for adequate temporal resolutions, and (ii) identify the appropriate discrete Hayami method for accurate channel routing.

3.2 Diffusion wave model with lateral inflow

3.2.1 General solution

When the friction slope of the water surface is much smaller than the channel bed slope, the linear convection-diffusion wave equation with uniformly distributed lateral inflow can be simplified as (Lighthill and Whitham, 1955; Moussa, 1996; Fan and Li, 2006)

\[
\frac{\partial Q}{\partial t} + C \frac{\partial Q}{\partial x} - D \frac{\partial^2 Q}{\partial x^2} = Cq
\]  

(3.1)
where $x$ is the downstream distance (m), $t$, time (s), $Q$, discharge ($m^3 \cdot s^{-1}$), $q$, lateral inflow rate per unit length ($m^2 \cdot s^{-1}$), $C = \frac{dQ}{dA}$, wave celerity ($m \cdot s^{-1}$), $D = \frac{Q}{2BS_f} \approx \frac{Q}{2BS_0}$, diffusive coefficient ($m^2 \cdot s^{-1}$), $A$, cross-sectional area ($m^2$), $B$, channel width at the water surface (m), $S_f$, friction slope, and $S_0$, channel bed slope.

For channel flow with steady-state initial conditions, the Hayami solution is (Moussa, 1996)

$$Q(x,t) = Q(x,0) + \Phi(t) + \left[ Q(0,t) - Q(0,0) - \Phi(t) \right] \otimes K(t), \quad (3.2)$$

where $\otimes$ is the convolution sign, and $K(t)$ is the Hayami kernel function defined as

$$K(t) = \frac{x}{2\sqrt{\pi Dt^3}} e^{-\frac{(x-\xi)^2}{4Dt}}, \quad (3.3)$$

and $\Phi(t)$ is a term related to the wave celerity and the lateral inflow

$$\Phi(t) = C \int_0^t [q(\tau) - q(0)] d\tau. \quad (3.4)$$

### 3.2.2 Discrete convolution solution with exact point kernel function values

When the channel inflow at the upper boundary is in a discrete form, Eq. (3.2) can be written as

$$Q^n = Q^0 + \Phi^n + \Delta t \sum_{j=0}^{n-1} \left[ Q^j - Q^0 - \Phi^j \right] K^{n-j} \quad (3.5)$$

where $\Delta t$ is the time step, $j$ and $n$ are the numbers of time steps, $Q^n$ and $Q^0$ are the channel flow rate at the outlet at time $t^n = n\Delta t$ and time 0, respectively, $\Phi^n$ and $\Phi^j$ are the discrete form of $\Phi(t)$ at time $n\Delta t$ and $j\Delta t$, respectively, $Q^j_u$ and $Q^0_u$ are the channel inflow at the upper boundary at time $j\Delta t$ and 0, respectively, and, $K^j$ is the discrete kernel function value at time step $j$. 

33
\[ K_j^i = \frac{x}{2\sqrt{\pi D(j\Delta t)^3}} e^{-\frac{(x-C(j\Delta t))^2}{4Dj\Delta t}}, \quad j=1,2,3,\ldots \]  

(3.6)

The term \( K_0 \) is omitted because its value is always zero.

Introducing the Courant number

\[ C_r = \frac{C\Delta t}{x} \]

and the Peclet number

\[ P_e = \frac{Cx}{2D} \]

we can rewrite Eq. (3.6) as

\[ K_j^i = \frac{1}{j\Delta t} \sqrt{\frac{P_e}{2\pi C_r}} e^{-\frac{(1-j\Delta t)^2 P_e}{2j^2C_r}}, \quad j=1,2,3,\ldots \]  

(3.7)

Eq. (3.5) is the discrete convolution solution using the exact point kernel function values \( K_j^i \) \((j=1, 2, 3, \ldots)\). The accuracy of the discrete convolution results depends on the time step size \( \Delta t \) and can be improved by decreasing \( \Delta t \) (Barlow et al., 2000). As the original kernel function is continuous and its integration is 1, the summation of its discrete form should also be unity for otherwise mass balance would not be maintained. However, when the time step becomes too large, there may be only a few discrete points left on the rising curve of the kernel function, causing the discrete integration of \( K_j^i \) to be larger or smaller than unity and thus an error in mass balance.

### 3.2.2.1 Determination of maximum time step

To determine a maximum time step, we first calculate the maximum \( K \) value, and estimate the time for \( K \) to rise from a small value, e.g., 0.1% of the maximum, to the maximum value.
From Eq. (3.3), we have
\[ K'(t) = -\frac{3}{2} \frac{x}{2\sqrt{\piDt^3}} e^{-\frac{(x-Ct)^2}{4Dt}} + \frac{x}{2\sqrt{\piDt^3}} e^{-\frac{(x-Ct)^2}{4Dt}} \left[ \frac{C(x-Ct)}{2Dt} + \frac{(x-Ct)^2}{4Dt^2} \right] \] (3.8)

Letting \( K'(t) = 0 \), and neglecting the negative solution, we obtain
\[ t_{km} = \frac{3D}{C^2} \left[ \sqrt{1 + \left( \frac{Cx}{3D} \right)^2} - 1 \right] \] (3.9)

where \( t_{km} \) is the time when \( K \) reaches its maximum value \( K_{\text{max}} \).

Expressing Eq. (3.9) in terms of \( C_r \) and \( P_e \) gives
\[ t_{km} = \frac{3\Delta t}{2C_rP_e} \left[ \sqrt{1 + \left( \frac{2}{3} \frac{P}{P_e} \right)^2} - 1 \right] \] (3.10)

or
\[ j_{km} \Delta t = \frac{3\Delta t}{2C_rP_e} \left[ \sqrt{1 + \left( \frac{2}{3} \frac{P}{P_e} \right)^2} - 1 \right] \] (3.11)

where \( j_{km} \) is the number of time steps when \( K \) reaches its maximum.

Eliminating the term \( \Delta t \) from both sides, we can simplify the above equation as
\[ j_{km} = \frac{3}{2C_rP_e} \left[ \sqrt{1 + \left( \frac{2}{3} \frac{P}{P_e} \right)^2} - 1 \right] \] (3.12)

Since a square root can be approximated as \( \sqrt{1 + x} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \ldots \), for \( |x| << 1 \),

if \( P_e << \frac{3}{2} \), we have
\[ \sqrt{1 + \left( \frac{2}{3} P_e \right)^2} \approx 1 + \frac{1}{2} \left( \frac{2}{3} P_e \right)^2 \]

Then Eq. (3.10) is reduced to

\[ t_{km} = \frac{P \Delta t}{3 C_r} = \frac{x^2}{6D} \]  

(3.13)

Similarly, if \( P_e \gg \frac{3}{2} \), we have

\[ \sqrt{1 + \left( \frac{2}{3} P_e \right)^2} = \frac{2}{3} P_e \sqrt{1 + \left( \frac{3}{2} P_e \right)^2} \approx \frac{2}{3} P_e \left[ 1 + \frac{1}{2} \left( \frac{3}{2} P_e \right)^2 \right] \]

Then Eq. (3.10) becomes

\[ t_{km} \approx \frac{3 \Delta t}{2 C_r P_e} \left[ \frac{2}{3} P_e + \frac{3}{4 P_e} - 1 \right] = \frac{\Delta t}{C_r} - \frac{3 \Delta t}{2 C_r P_e} = \frac{x}{C} - \frac{3D}{C^2} \]  

(3.14)

where the term \( \frac{x}{C} \) is the travel time of the kinematic wave.

For a kinematic wave, \( D = 0 \), and Eq. (3.14) becomes

\[ t_{km} = \frac{x}{C} \]  

(3.15)

and Eq. (3.5) is simplified to

\[ Q^a = Q^0 + \Phi^a + Q^k - Q^0 - \Phi^k \]  

(3.16)

since \( K_{km} = 1/\Delta t \) and all other \( K^j = 0 \) (\( j = 1, 2, \ldots, j_{km}-1, j_{km}+1, \ldots \)). If there is no lateral inflow, Eq. (3.16) is reduced to

\[ Q^a = Q^k \]  

(3.17)

recovering the characteristic solution to the kinematic-wave model.
Letting $K(t) = 0.001K_{\text{max}}$ and using Newton’s method, we can obtain $t_f$, the time when $K$ reaches 0.1% of $K_{\text{max}}$ on the rising limb. The maximum time step for channel routing using Eq. (3.5) can then be estimated as

$$\Delta t_{\text{max}} = \frac{t_{100} - t_f}{N}$$

(3.18)

where $N$ is the number of time steps on the rising limb and can be varied to attain the desired resolution.

Higdon (2002) obtained satisfactory results with a discrete spatial interval close to the standard deviation of the Gaussian kernel function ($N \geq 3$). The principle of discrete convolution in a space domain is the same as in a time domain. For a greater accuracy, Merkel (2002) suggested the use of 10 time steps to assure an adequate resolution of the rising limb of a hydrograph.

### 3.2.3 Discrete convolution solution with average kernel function values

In the unit-step response method, the average of $K^j$ is calculated from time $t$ to $t+\Delta t$ (Dooge, 1959; Chow et al., 1988, p. 207; Hantush et al., 2002; Hantush, 2005). As a replacement of the exact point kernel function value $K^j$, the center average of $K^j$ around a discrete point $j$ can be calculated using the following equation

$$K^j \approx \frac{1}{\Delta t} \int_{t-j/2}^{t+j/2} K(\tau) \, d\tau$$

$$= \frac{1}{\Delta t} \left[ \int_{0}^{t+j/2} K(\tau) \, d\tau - \int_{0}^{t+j/2} K(\tau) \, d\tau \right] \quad \text{for} \quad j = 1, 2, \ldots, n$$

$$= \frac{1}{\Delta t} \left[ S(t+j/2) - S(t-j/2) \right]$$

and

$$K^0 = \frac{2}{\Delta t} S(t/2) \quad \text{for} \quad j = 0$$

(3.19)
where \( j \) is the time step number, \( t^j \) is the time (s) at step \( j \), and \( S(t) \) is the storage function (Dooge, 1959; Fan and Li, 2006), which is the same as the unit-step response function (Chow et al., 1988, p. 205) and is given by (Ogata and Banks, 1961; Fan and Li, 2006)

\[
S(t) = \int_0^t K(\tau) \, d\tau = \frac{1}{2} \left[ \text{erfc} \left( \frac{x - Ct}{2\sqrt{Dt}} \right) + e^{\frac{c}{2}} \text{erfc} \left( \frac{x + Ct}{2\sqrt{Dt}} \right) \right]
\]  

(3.20)

Substituting \( \overline{K}^j \) for \( K^j \) in Eq. (3.5), we have

\[
Q^n = Q^n_0 + \Phi^n + \sum_{j=0}^{n-1} (Q^n_u - Q^n_0 - \Phi^n) \left[ S \left( t^{\frac{n-j+\frac{1}{2}}{2}} \right) - S \left( t^{\frac{n-j-\frac{1}{2}}{2}} \right) \right] + (Q^n_u - Q^n_0 - \Phi^n) S \left( t^{\frac{1}{2}} \right)
\]

(3.21)

Eq. (3.21) is the discrete convolution solution to Eq. (3.1) using the average kernel function values \( \overline{K}^j \) (\( j = 0, 1, 2, 3, \ldots \)). Note the difference between the average kernel function and the point kernel function at time zero: the value of the former at time zero, \( \overline{K}^0 \), may not be zero. The use of the average kernel function reserves mass balance better since the storage function approaches unity for sufficiently large time (Fan and Li, 2006).

Similarly, instead of using an inflow at time \( t \), we may use the average of the inflow between two half time steps in Eq. (3.21) (Hayami, 1951). However, averaging both the inflow and kernel functions for the convolution calculation may lead to further wave attenuation and is not discussed in this study.

When the discrete convolution solution is used to calculate the discharge, the time step in the kernel function must be the same as in the input discharge. If the time step is smaller than \( \Delta t_{\text{max}} \) in Eq. (3.18), Eqs. (3.5) and (3.6) can be used to calculate channel flow. Otherwise, we either interpolate the known discharges (Merkel, 2002) or use Eqs. (3.20) and (3.21) to calculate channel flow, to assure mass balance and adequate accuracy.
### 3.2.4 Integration error of the discrete kernel function

For a constant inflow with homogenous initial condition and with no lateral inflow within the stream reach, the relative mass balance error of outflow resulting from the discrete kernel function is the same as the integration error of the discrete kernel function

\[
\Delta M = \left(1 - \Delta t \sum_{j=1}^{\infty} \frac{K^{j-1} + K^j}{2}\right) \times 100\%
\]

\[
= \left[1 - \Delta t \left(\frac{K^0}{2} + \sum_{j=1}^{\infty} \frac{K^j}{2}\right)\right] \times 100\%
\]

\[
= \left[1 - \Delta t \left(\frac{K^0}{2} + \sum_{j=1}^{\infty} \frac{K^j}{2}\right)\right] \times 100\%
\]

For channel routing with lateral inflow or when channel inflow at upper boundary varies with time, the mass balance error may be different from \(\Delta M\). This can be seen from Eq. (3.5) where the term \(Q_u^j - Q_u^0 - \Phi^j\) is temporally variable.

### 3.2.5 Channel water storage

To track the channel water balance, we first calculate the initial cross-section area for the steady-state channel inflow and outflow using Manning’s equation, and estimate the initial channel water storage (\(S^0, m^3\)) by multiplying the average cross-section area by the channel length. The storage \(S^j (m^3)\) at time \(t^j\) is calculated by (Todini, 2007)

\[
S^j = S^{j-1} + V_{in} - V_{out}
\]

\[
= S^{j-1} + \left(\frac{Q_u^j + Q_{u-1}^j}{2} + \frac{q^j + q_{-1}^j}{2} \chi\right) \Delta t - \left(\frac{Q_u^j + Q_{u-1}^j}{2}\right) \Delta t, \ j = 1, 2, 3, \ldots
\]
3.3 A numerical experiment

For a numerical experiment, we assume a rectangular channel with width $B=50$ m, length $L=30000$ m, Manning’s roughness $n=0.035$, bed slope $S_0=0.002$, and the following initial and boundary conditions

at $t = 0$,

$$Q(x,0) = 0, \quad x \geq 0,$$  \hspace{1cm} (3.24)

at $x = 0$,

$$Q(0,t) = 0, \quad 0 < t < 3600$$
$$Q(0,t) = 80 \times \frac{t - 3600}{3600}, \quad 3600 \leq t < 7200$$
$$Q(0,t) = 80, \quad 7200 \leq t \leq 72000$$
$$Q(0,t) = 80 \times \frac{75600 - t}{3600}, \quad 72000 < t < 75600$$
$$Q(0,t) = 0, \quad 75600 \leq t$$  \hspace{1cm} (3.25)

where $t$ is in s, $x$ in m, and $Q$ in m$^3$ s$^{-1}$.

3.3.1 Comparison of discrete convolution solutions with Muskingum-Cunge solution

Fig. 3.1 shows the results from the discrete convolution solutions and the constant-parameter Muskingum-Cunge (CPMC) method (Ponce and Chaganti, 1994) for a time step of 600 s. Both discrete convolution solutions, with point or average kernel function values, compare well with the CPMC solution. The differences (maximum 0.5%) occur only along the rising and falling limbs (Fig. 3.1).

3.3.2 Discrete convolution solution with point kernel function values

When the time step is small, the discrete Hayami convolution method with point kernel function values gives satisfactory results (Fig. 3.2). For the hypothetical case described by Eqs. (3.24) and (3.25), the difference between the results with a time step of 600 s or 60 s is negligible. The
respective $N$ values are 8 and 80, with the rising limb of the kernel function starting at $t = 12,425$ s and reaching the maximum at $t = 17,200$ s (Fig. 3.3). For the time step of 3600 s, however, the calculated peak discharge is 78.7 m$^3$ s$^{-1}$, 1.7% lower than the theoretical value of 80.0 m$^3$ s$^{-1}$. This error is caused by the inadequate resolution of the discrete kernel function. When the time step is increased to 3600 s, $N = 1.33$, and the integration of the discrete Hayami kernel function is not unity (Fig. 3.3), resulting in the mass balance error.

3.3.3 Discrete convolution solution with average kernel function values

The use of average kernel function values always preserves mass balance. The simulated peak discharge is the same for different sizes of time steps (Fig. 3.4) and the integration of the discrete kernel function is always unity (Fig. 3.5).

3.3.4 Comparison of different calculation methods for discrete kernel function

For the relatively small time step of 60 s, the calculated discrete kernel functions using the point- or average values are almost the same. For the time step of 3600 s, the two calculation methods lead to different results (Fig. 3.6a). The point $K$ values do not coincide with the average $K$ values, indicating their failure to represent the average $K$ values of the analytical solution within the corresponding time step. The discrete integration of the point-kernel function is not unity, resulting in mass balance error.

3.3.5 Kernel function and temporal resolution

When the discrete Hayami kernel function with average values is used, the integration error of the kernel function in calculated outflow is independent of the size of time step (Fig. 3.6b, c). This is because the storage function always reaches unity given sufficiently long time. When the discrete kernel function with point values is used, the temporal resolution will affect the integration of the kernel function, and hence the accuracy of mass balance. The relative
integration error $\Delta M$ ranges from −9.4% to 15% for 1.14–1.73 time steps on the rising limb, and is less than 0.38% when the rising limb is longer than 1.77 time steps or for $\Delta t < 45$ min (Fig. 3.6b, c).

### 3.3.6 Computing cost

On a Pentium 4 PC, the CPU time for running the FORTRAN program with the discrete Hayami convolution solution using the point $K$ values or the average $K$ values is 0.125 s for 2000 time steps, the same as for running the CPMC solution.

### 3.4 Applications

We chose two streams in the Pacific Northwest for our application, the Asotin Creek, WA, and the Clearwater River, ID (Fig. 3.7).

#### 3.4.1 Asotin Creek, WA

Asotin Creek is located in eastern Washington State, USA (Fig. 3.7a). It joins the Snake River at Asotin, WA. The watershed is typical of many watersheds in the western USA, with high-elevation snowmelt providing much of the runoff. Streamside agriculture and domestic users are dependent on the streamflow for livestock, irrigation, and human consumption. Much of the development has been, and continues to be, in the flood plains. Therefore, understanding fluctuations in streamflow and peak flow responses to high melt rates, often associated with rainfall in the mountains, is important (Goodwin et al., 1997). This importance will likely increase with the projected increase in large storm events in the coming century (USDI, 2011).

There are two real-time gauging stations that are 16.2 km apart, the upstream US Geological Survey 13334450 (elevation 552 m, inflow), and the downstream Washington State Department of Ecology 35D100 (elevation 317 m, outflow) (Fig. 3.7b). The distributed lateral inflow is typically positive during low streamflow periods and negative at peak flows, suggesting active
surface water and groundwater interactions. The lateral inflow was estimated based on water balance and the difference of hydraulic heads between stream water and a calibrated level of groundwater in the underlying aquifer following McDonald and Harbaugh (1988) and Chen and Chen (2003) and was assumed to be uniformly distributed along the channel between the two gauging stations. All the observed streamflow data are recorded in a 15-min interval and the hydrographs are shown in Fig. 3.8a. Figure 3.8b shows the observed temperature from the 35D100 station, and the observed temperature and precipitation at the US Department of Agriculture SNOTEL station at Sourdough Gulch (elevation 1220 m).

The simulated peak discharge is 2.0% lower than the observed value, with a root-mean-square error (RMSE) of 0.24 m$^3$ s$^{-1}$, when using the point kernel function values for the discrete Hayami convolution solution, and is 0.19% higher than the observed value, with a RMSE of 0.15 m$^3$ s$^{-1}$, when using the average kernel function values (Fig. 3.8a).

The relatively larger errors in simulating the peak discharge and total volume using the point kernel function values for the discrete Hayami solution are due to the error of the integration of the discrete kernel function. For a 15-min time step, there are only 1.65 time steps for the rising limb of the kernel function (Fig. 3.9a) of which the integration over time is 0.987, or a $-1.3\%$ error. The solution becomes unstable as the size of the time step is further increased (Fig. 3.9b, c). If $\Delta t \leq 14$ min, the number of time steps $N \geq 1.76$, and the integration error ($\Delta M$) of the discrete $K$ will be smaller than 0.62%.

A comparison of the discrete Hayami solution with the CPMC method shows that the differences between the Hayami solution with average kernel function values and the CPMC method fluctuate closely to zero, with an RMSE of 0.016 m$^3$ s$^{-1}$, whereas the differences between the Hayami solution with point kernel function values and the CPMC method tend to be
substantially less than zero, though also with fluctuations, with an RMSE of 0.17 m$^3$ s$^{-1}$ (Fig. 3.10). The simulated channel water storage using the average kernel function values for the discrete Hayami solution is close to that by the CPMC method, while the simulated water storage using the point kernel function values for the discrete Hayami solution increases unrealistically when the channel discharge continuously decreases, due to the mass balance error in calculating the outflow (Fig. 3.11).

This computation took a Pentium 4 PC 0.156 s when the discrete Hayami solution with either point or the average method was used, and 0.140 s when the CPMC method was used.

### 3.4.2 Clearwater River, ID

The Clearwater River is the second largest tributary to the Snake River in the Columbia Basin, US Pacific Northwest (Fig. 3.7a). Its middle fork has been listed as one of the nation’s wild and scenic rivers in the Wild & Scenic Rivers Act passed in 1968. The recovery of salmon and steelhead habitat in Clearwater River has been an important management goal for decades (ICTRT, 2007). Adequate flow routing is crucial to understanding streamflow regime in relation to salmon migration and seasonality. There are two USGS gauging stations for the Clearwater River between Peck and Spalding, in north central Idaho State. The main tributary in this reach, the Potlatch River, joins the Clearwater River at river mile 14.9 (24 km) and is also gauged by the USGS (Table 3.1). Upstream of the Clearwater River near Peck is a regulated reservoir, the Dworshak Reservoir (Fig. 3.7c). The active operation of the reservoir generates frequent waves. For our application, we selected a single-peak wave during Feb 8–10, 2009 (Fig. 3.12).

#### 3.4.2.1 Potlatch River, ID

For the relatively short distance (2 mi or 3.2 km) between the USGS gauging station on Potlatch River and its confluence with Clearwater River, the calculated maximum value of the discrete
kernel function is $7.94 \times 10^{-4}$, at time 879 s, or 14.7 min, less than the temporal interval of 15 min for the observation data (Fig. 3.13a). With this temporal interval, the number of time steps on the rising limb of the kernel function is 0.7 and the integration error of the discrete kernel function using the point value is 6.6% (Fig. 3.13b, c). The integration error would be negligible if $\Delta t \leq 7$ min or $N \geq 1.52$, positive if $7 < \Delta t \leq 18$ min, and always negative if $\Delta t > 18$ min. With further increased $\Delta t$, the integration error would increase until it approached $-100\%$, when all the effective $K$ values were missed. The integration error of the discrete kernel function using the average values is negligible, regardless of the size of time step, as demonstrated in the previous case.

The simulated outflow using discrete Hayami solution with point and average kernel function values and using the CPMC method are shown in Fig. 3.14. The result from the discrete Hayami convolution solution using the average kernel function values is very close to that from the constant-parameter Muskingum-Cunge method, and their difference is small fluctuations about zero (Fig. 3.15). Comparing with the CPMC method, the error of the results by the discrete Hayami convolution solution using the point $K$ values is positive when the inflow is greater than the initial condition, and negative when the inflow is less than the initial condition, and grows with time (Fig. 3.15).

The simulated channel storage from the discrete Hayami convolution solution using average $K$ values and from the CPMC method are essentially identical (Fig. 3.16). Because of the error in the simulated discharge from the discrete Hayami convolution solution using the point $K$ values, the simulated channel storage would increase unrealistically with time even when the channel flow, and thus the channel stage, are decreasing (Figs. 3.14 and 3.16).
3.4.2.2 Clearwater River, ID

The upper reach of the Clearwater River flows from river mile 37.4 to 14.9 (60.2–24.0 km). With more than five steps (\(\Delta t = 15 \text{ min}\)) on the rising limb of the Hayami kernel function, the point \(K\) values are close to the average \(K\) values (Fig. 3.17a) and the integration error of the discrete kernel function is negligible.

Figs. 3.17b and 3.17c show that if the number of time steps on the rising limb of the kernel function \(M \geq 1.75\) or \(\Delta t \leq 50\) min, the integration error of the discrete kernel function using point kernel function values would be less than 0.19%. For \(\Delta t \leq 56\) min, \(N \geq 1.56\), and the integration error of the discrete \(K\) is less than 0.58%. For \(\Delta t\) ranging 57–66 min, \(N\) ranges 1.53–1.32, and the integration error of discrete \(K\) varies from −0.18% to −2.15%. For \(\Delta t\) ranging 67–70 min, \(N\) ranges 1.3–1.25, and the integration error of discrete \(K\) increases from 0.25% to 3.74%. The integration of the discrete \(K\) using the average \(K\) values remains unity regardless of the size of time step. The simulated outflow and channel storage by using these two methods is almost the same (Fig. 3.18).

The lower reach of the Clearwater River is from river mile 14.9, the junction with its tributary, the Potlatch River, to river mile 11.6. For this 3.3-mi reach, there are only 1.3 time steps on the rising limb of the Hayami kernel function (Fig. 3.19a), with an error of −2.3% in integrating the discrete kernel function using point \(K\) values. If the number of time steps on the rising limb of the kernel function \(N \geq 1.75\) or \(\Delta t \leq 11\) min, the integration error of the discrete kernel function using point kernel function values would be less than 0.29% (Fig. 3.19b, c). For \(\Delta t\) ranging 12–17 min, \(N\) ranges 1.61–1.13, and the integration error of discrete \(K\) changes from −0.88% to −2.3%. For \(\Delta t\) ranging 18–37 min, \(N\) ranges 1.07–0.52, and the integration error of discrete \(K\) is between 0.41% and 20%. For \(\Delta t \geq 38\) min, \(N \leq 0.51\), and the integration of discrete \(K\) is much
less than unity. The integration error of the discrete $K$ using the average $K$ values is negligible regardless of the size of time step.

Though the resultant error in mass balance for the simulated outflow is small, and the simulated flow only differs during peak flow time from that by using the average $K$ values, the effect on the simulated channel water storage is significant (Fig. 3.20). The differences between the simulated and the observed discharge are due to (i) the neglecting of the rather small amount of temporally variable lateral inflow or outflow and (ii) the use of the linear diffusion-wave model that does not account for the steepening effect of the wave. The former can be improved with available data, and the second by differentiating the non-linear system to parallel linear systems (Becker, 1976), using a multi-linear model (Perumal, 1994; Camacho and Lees, 1999; Perumal et al., 2007) or the modified variable-parameter Muskingum-Cunge method (Ponce and Chaganti, 1994), or dividing the channel reach to sub-reaches and applying the discrete Hayami convolution solution to each sub-reach.

3.5 Summary and Conclusions

The Hayami convolution solution can be used in constant-parameter diffusion-wave channel routing with distributed lateral inflow. However, the convolution of channel inflow with the continuous Hayami kernel function has analytical solutions only for simplified cases and is costly when solved using numerical integration for watershed modeling that often involves hundreds of reaches. It is generally more efficient to use the discrete Hayami convolution.

The Hayami kernel function itself is continuous over time from $0$ to $+\infty$, and its integration over this range is unity. Yet attention should be paid to the size of the time steps when we use its discrete exact point values in replacement of the continuous function. When the rising limb of the Hayami kernel function is shorter than 1.8 time steps in duration, the integration of the
discrete Hayami kernel function using the point kernel function values is hardly unity, which would lead to mass balance error for channel routing. Hence, this solution applies only to cases with rising limb of the kernel function longer than 1.8 time steps, as demonstrated in the hypothetic numerical experiment with small time step sizes or a relatively larger time step size for a longer channel (upper reach of Clearwater River). When this criterion is not met, there is a high risk of simulated mass balance failure. This has been shown in our case applications for Asotin Creek, Potlatch River, and lower reach of Clearwater River. In Asotin Creek and lower reach of Clearwater River case applications, with 1.65 and 1.3 time steps on the rising limb of their kernel functions, respectively, the errors for simulated outflow were not too large, but the errors of the simulated water storage were significant and increased with time, as a result of the accumulation of mass balance error. In Potlatch River case application, with only 0.7 time step on the rising limb of the kernel function, the error in simulated outflow was significant and increased with time.

Another approach is to use the center-averaged kernel function values. Our hypothetic example and case applications to Asotin Creek, Potlatch River, and the upper and lower reaches of Clearwater River showed that the integration of the discrete kernel function using the averaged kernel function values is always unity, regardless of temporal resolution. Therefore, we recommend the use of this method to preserve the mass balance of channel flow when the rising limb of the kernel function is short, or in watershed channel routing with numerous channel reaches, for which the kernel functions may be different.
3.6 Tables and Figures
Table 3.1. USGS gauging stations on Clearwater River near Peck and Spalding, ID.

<table>
<thead>
<tr>
<th>USGS station</th>
<th>River</th>
<th>Location</th>
<th>River mile (River km)</th>
<th>Drainage Area mi² (km²)</th>
<th>Elevation m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clearwater</td>
<td>Near Peck, ID</td>
<td>37.4 (60.2)</td>
<td>7976 (20,660)</td>
<td>930</td>
<td></td>
</tr>
<tr>
<td>Clearwater</td>
<td>At Spalding, ID</td>
<td>11.6 (18.7)</td>
<td>9283 (24,040)</td>
<td>770</td>
<td></td>
</tr>
<tr>
<td>Potlatch</td>
<td>Below Little Potlatch Cr. near Spalding, ID</td>
<td>2.0 (3.2)</td>
<td>583 (1510)</td>
<td>845</td>
<td></td>
</tr>
</tbody>
</table>

<sup>a</sup> Inflow station  
<sup>b</sup> Outflow station
Figure 3.1. Comparison of discrete convolution method and the constant-parameter Muskingum-Cunge (CPMC) method for a time step of 600 s. Note that simulated outflows from the discrete Hayami solution and the CPMC method largely overlap, and therefore only the differences between the two methods are shown.
Figure 3.2. Discrete Hayami convolution solutions using point kernel function values for different sizes of time step. The solutions for 600-s and 60-s time steps largely overlap. For the time step of 3600 s, the calculated peak discharge is 1.7% lower than the theoretical value.
Figure 3.3. Discrete Hayami kernel functions with point values for different sizes of time step.
Figure 3.4. Discrete Hayami convolution solution using average kernel function values. The calculated peak discharge is not affected by the size of time step. The solutions for the 60-s and 600-s time steps largely overlap, and the calculated peak discharge is the same for all three time steps.
Figure 3.5. Discrete Hayami kernel function from using average values for different sizes of time step.
Figure 3.6. Comparison of discrete Hayami kernel functions using point- or average values (a), and the effect of the size of time step (b) and the number of time steps (c) on the integration error.
Figure 3.7. (a) Location of Asotin Creek and Clearwater River basins, northwestern US, (b) Asotin Creek basin, and (c) Lower Clearwater River basin.
Figure 3.8. (a) Observed and simulated hydrographs for Asotin Creek, WA, in response to the rainfall and snowmelt events during December 1–16, 2007, and (b) observed temperature and precipitation for the same period.
Figure 3.9. Comparison of discrete Hayami kernel functions using point- or average values (a), and the effect of the size of time step (b) and the number of time steps (c) on the integration error in the discrete Hayami kernel function for channel flow routing, Asotin Creek, WA, December 1–16, 2007.
Figure 3.10. Differences in simulated discharges between the Hayami discrete convolution methods with point- or average kernel function values and the constant-parameter Muskingum-Cunge (CPMC) method, Asotin Creek, WA, December 1–16, 2007.
Figure 3.11. Simulated channel water storage by different methods, Asotin Creek, WA, December 1–16, 2007. Note that the simulated storages using the discrete Hayami solution with average kernel function values and using the constant-parameter Muskingum-Cunge method largely overlap, thus only the former is shown.
Figure 3.12. Observed hydrographs at the three gauging stations on Clearwater River and its tributary Potlatch River. Note different discharge scales used.
Figure 3.13. Comparison of discrete Hayami kernel functions using point- or average values (a), and the effect of the size of time step (b) and the number of time steps (c) on the integration error in the discrete Hayami kernel function for channel flow routing, Potlatch River, ID, February 8–10, 2009.
Figure 3.14. Observed inflow and simulated outflow for the Potlatch River (with the assumption of no lateral inflow), February 8–10, 2009. The results from the discrete Hayami solution using the average kernel function values and the CPMC method largely overlap.
Figure 3.15. Differences in discharges between the discrete Hayami convolution solution and the CPMC method, Potlatch River, ID, February 8–10, 2009.
Figure 3.16. Simulated channel storage using different methods, Potlatch River, ID, February 8–10, 2009. Note that the simulated storages from the discrete Hayami solution using the average kernel function values and the CPMC method largely overlap.
Figure 3.17. Comparison of discrete Hayami kernel functions with point- or average values (a), and the effect of the size of time step (b) and the number of time steps (c) on the integration error in the discrete Hayami kernel function for channel flow routing, upper reach, Clearwater River, ID, February 8–10, 2009.
Figure 3.18. Simulated outflow and channel storage using different methods for the upper reach of the Clearwater River, ID, February 8–10, 2009. Note that the simulated outflow (or storage) from the discrete Hayami solution using the point- or average kernel function values largely overlap.
Figure 3.19. Comparison of discrete Hayami kernel functions using point- or average values (a), and the effect of time step (b) and number of time steps (c) on integration errors in the discrete Hayami kernel function for channel routing, lower reach, Clearwater River, ID, February 8–10, 2009.
Figure 3.20. Simulated outflow and channel storage using different methods, lower reach, Clearwater River, ID, February 8–10, 2009.
4.0 Abstract

Channel routing is crucially important in flood forecasting and watershed modeling. Typically, the upstream channel inflow is more critical in flood routing, and the lateral inflow is more important for watershed modeling. The accuracy of lateral inflow estimation can substantially affect the accuracy of the channel routing in watershed modeling. The general constant-parameter Muskingum-Cunge (CPMC) method is second-order accurate and easy to implement. With specific discretizations such that the temporal and spatial intervals maintain a unique relationship, the CPMC method can be of third-order accuracy. In this study, we derived the average lateral inflow term in the second- and third-order accuracy CPMC method for channel routing. The derived equations indicated that for spatially and temporally variable lateral inflow, the effect of lateral inflow on simulated discharge was affected not only by spatial and temporal discretizations, the value of lateral inflow, its variation with space and time, but also by wave celerity and diffusion coefficient. Comparison of the CPMC solution with the analytical solution showed that both the second- and third-order accuracy schemes were generally more accurate than the simplified method by which spatial derivatives of lateral inflow were ignored. The accuracy of the second-order CPMC method increased with decreasing time-step sizes for relatively large time steps, and remained nearly constant for smaller time steps. For small time

1 This chapter will be submitted for publication: Wang, L., Lapin, S., Wu, J.Q., Elliot, W.J., Fiedler, F.R. Accuracy of Muskingum-Cunge method diffusion-wave channel routing with lateral inflow. J. Hydrol.
steps, the third-order accuracy CPMC method led to higher accuracy than the second-order scheme even when the third-order accuracy criterion was not fully met. For large time steps, the temporal and spatial discretization of the third- and second-order scheme became the same, but the third-order scheme yielded higher accuracy than the second-order scheme because of the third-order accurate estimation of the lateral inflow term used in the CPMC method.

4.1 Introduction

Channel upstream inflow is usually the most important component for flood routing. In watershed modeling, however, channel water often comes from lateral inflow. As in the Water Erosion Prediction Project (WEPP) model, water generated from a hillslope (surface runoff, subsurface lateral flow, and groundwater base flow) may enter a channel as upstream inflow when the hillslope is at the top of the channel, or as lateral inflow when the hillslope is on the side of the channel (Fig. 4.1). A hillslope can be at the top of a channel only in cases of 1st-order channels, and would otherwise be on the side of the channel, with the top of the channel being upstream channels or an impoundment (Flanagan and Livingston, 1995). In addition, the gain or loss of the stream water by precipitation, infiltration, and evapotranspiration is often included in the lateral inflow term. In commonly used numerical channel routing methods, e.g., the Muskingum-Cunge or the kinematic-wave method, we need to calculate the average lateral inflow in the channel routing equation. The order of accuracy of the average lateral inflow term can be a dominant factor affecting the accuracy of the numerical channel routing in watershed simulations.

Price (2009) developed a second-order accurate nonlinear diffusion-wave scheme considering uniform lateral inflow and solved it using the Newton-Raphson iterative method. The author also analyzed the effect of bed slope on the accuracy and found the accuracy decreased with
decreasing bed slope. Barry and Bajracharya (1995) showed that for channel routing without lateral inflows, when the time step $\Delta t$ and the space interval $\Delta x$ maintained a certain relationship so that the Courant number is 0.5, the Muskingum-Cunge method was third-order accurate. For constant-parameter diffusion-wave channel flows without lateral inflow, Bajracharya and Barry (1997) derived a relationship of spatial and temporal steps of $\Delta t = \frac{C \Delta x - 2D}{C^2}$, where $C$ denotes kinematic celerity and $D$ diffusion coefficient, to assure a second-order accuracy of Muskingum-Cunge scheme, a relationship of $\Delta t = \frac{1}{C} \sqrt{\Delta x^2 - \frac{12D^2}{C^2}}$ or $\Delta x = \sqrt{C^2 \Delta t^2 + \frac{12D^2}{C^2}}$ for third-order accuracy, and fixed $\Delta t = \frac{2\sqrt{3}D}{C^2}$ and $\Delta x = \frac{2\sqrt{6}D}{C}$ for fourth-order accuracy. Szel and Gaspar (2000), without considering lateral inflow, related the temporal and spatial intervals to the Courant number $C_r = \frac{C \Delta t}{\Delta x}$ and Peclet number $P_e = \frac{C \Delta x}{2D}$, discussed their effect on the stabilities of the Muskingum-Cunge scheme, and found that the relationship of the spatial and temporal steps required for the third-order Muskingum-Cunge method can be simplified to a dimensionless equation $C_r^2 + \frac{3}{P_e^2} - 1 = 0$.

In addition to the relationship between $\Delta x$ and $\Delta t$, Moramarco et al. (1999) reported that the choice of reference discharge, which is used to calculate the kinematic celerity and the diffusion coefficient, can also affect the accuracy of the channel routing with lateral inflow. By testing the channel routing without the upstream inflow, they found that the error in channel routing changed with reference discharge and bed slope. For a channel with a relatively gentle slope, such as 0.0001, the selected reference discharge should be larger for a better accuracy; for a
channel with a rather steep slope, e.g., 0.01, the accuracy of the channel routing was not sensitive
to the reference discharge.

The lateral inflow in a channel routing equation was usually treated as concentrated or
uniformly distributed for simplicity (Chow et al., 1988; Fan and Li, 2006). When lateral inflow is
spatially and temporally variable, its effect on accuracy of numerical channel routing has not
been discussed. In this study, we will (i) derive a second- and third-order accurate representation
for the lateral inflow term used in the constant-parameter Muskingum-Cunge (CPMC) method
for channel routing, (ii) compare the results from the third- and the second-order accuracy
CPMC methods with analytical solution, and analyze the effect of the time-step size on the
accuracy of the CPMC solution.

4.2 Methods

The constant-parameter diffusion-wave equation with lateral inflow can be simplified as
(Lighthill and Whitham, 1955; Bajracharya and Barry, 1997; Fan and Li, 2006; Price, 2009)

\[ \frac{\partial Q}{\partial t} + C \frac{\partial Q}{\partial x} - D \frac{\partial^2 Q}{\partial x^2} = Cq \quad (4.1) \]

where \( Q = Q(x,t) \) is discharge (\( m^3 \ s^{-1} \)), \( x \) is downstream distance (m), \( t \) is time (s), \( q = q(x,t) \) is
lateral inflow rate per unit length (\( m^2 \ s^{-1} \)), with positive \( q \) representing flow into, and negative \( q \)
flow out of, the channel, \( C = \frac{dQ_R}{dA} \) is kinematic wave celerity (m s\(^{-1} \)), and \( D = \frac{Q_R}{2BS_f} \approx \frac{Q_R}{2BS_0} \) is
the diffusion coefficient (m\(^2\) s\(^{-1} \)) where \( Q_R \) is the reference discharge, \( A \) the cross-sectional area
(\( m^2 \)), \( B \) the channel width at the water surface (m), \( S_f \) the friction slope, and \( S_0 \) the channel bed
slope.
4.2.1 Third-order accuracy CPMC method

The Muskingum-Cunge method solving Eq. (4.1) numerically is (Chow et al., 1988; Ponce, 1995; Bajracharya and Barry, 1997; Szel and Gaspar, 2000)

\[
Q_{i+1} = C_1 Q_i + C_2 Q_i^{j+1} + C_3 Q_i + C_4 q_i^{j+1} \Delta x
\]  

(4.2)

where the Muskingum-Cunge coefficients are given by

\[
C_1 = \frac{\Delta x + C \Delta t - \frac{2D}{C}}{\Delta x + C \Delta t + \frac{2D}{C}}
\]  

(4.3)

\[
C_2 = \frac{-\Delta x + C \Delta t + \frac{2D}{C}}{\Delta x + C \Delta t + \frac{2D}{C}}
\]  

(4.4)

\[
C_3 = \frac{\Delta x - C \Delta t + \frac{2D}{C}}{\Delta x + C \Delta t + \frac{2D}{C}}
\]  

(4.5)

and

\[
C_4 = \frac{2C \Delta t}{\Delta x + C \Delta t + \frac{2D}{C}} = 1 - C_3
\]  

(4.6)

and \(q_i^{j+1}\) is the average lateral inflow. For uniformly distributed lateral inflow, it was calculated as \(q_i^{j+1} = \frac{q_i + q_i^{j+1}}{2}\) (Chow et al., 1988; Appendix A).

The CPMC method is second-order accurate without restrictions on temporal and spatial discretizations (Appendix A). To achieve the third-order accuracy without changing the representations of the Muskingum-Cunge coefficients, the spatial and temporal intervals must
satisfy the following relationships (Bajracharya and Barry, 1997; Szel and Gaspar, 2000; Appendix A)

\[
\Delta x = \sqrt{C^2 \Delta t^2 + \frac{12D^2}{C^2}}
\]  
(4.7)

for a given \( \Delta t \), or

\[
\Delta t = \frac{1}{C} \sqrt{\Delta x^2 - \frac{12D^2}{C^2}}
\]  
(4.8)

with \( \Delta x \) fixed. Eqs. (4.7) and (4.8) are equivalent to the following dimensionless equation (Szel and Gaspar, 2000)

\[
C_r^2 + \frac{3}{P_c^2} - 1 = 0
\]  
(4.9)

Hence, for a diffusion wave with \( P_c > \sqrt{3} \), the simulated outflow is of a higher-order accuracy if

\[
C_r = \sqrt{1 - \frac{3}{P_c^2}}
\]  
(4.10)

The third-order accuracy average lateral inflow can then be calculated as (Appendix A)

\[
\bar{q}_{i+1} = q_i \left[ 1 + \frac{1}{2} q_i \Delta t + \frac{1}{2} q_x \left( \Delta x + \frac{2D}{C} \right) + \frac{1}{6} q_{xx} \left( \Delta x^2 - \frac{1}{3} C^2 \Delta t^2 + \frac{2D\Delta x}{C} \right) \right]
+ \frac{1}{4} q_{xx} \left( \Delta x + \frac{1}{3} C \Delta t + \frac{2D}{C} \right) \Delta t + \frac{1}{4} q_{xxx} \left( \Delta x^2 - \frac{1}{3} C^2 \Delta t^2 + \frac{2D\Delta x}{C} \right) D \Delta t + \frac{1}{6} q_{xxxx} D^2 \Delta t^2 + \frac{1}{6} q_{xxx} D^2 \Delta t^2
\]  
(4.11)

We can also show that, for the second-order accuracy CPMC (Appendix A)

\[
\bar{q}_{i+1} = q_i \left[ 1 + \frac{1}{2} q_i \Delta t + \frac{1}{2} q_x \left( \Delta x + \frac{2D}{C} \right) \right] + \frac{1}{2} q_{xx} \Delta t
\]  
(4.12)
Eqs. (4.11) and (4.12) show that, $\bar{q}^{i+1}_{i+1}$ depends not only on lateral inflow and its spatial and temporal variation as well as the spatial and temporal discretization, but also on wave celerity and the diffusion coefficient of the channel flow.

If the spatial variation of lateral inflow is negligible, the third- and second-order accuracy average lateral inflow can also be estimated from a discrete dataset (Appendix A), i.e.,

$$\bar{q}^{i+1}_{i+1} = \begin{cases} 
- q^{i+2}_{i+1} + 8q^{i+1}_{i+1} + 5q^j + O(\Delta t^3) & \text{for } j = 0 \\
5q^{i+1}_{i+1} + 8q^j - q^j - 1 + O(\Delta t^3) & \text{for } j = 1, 2, \ldots 
\end{cases}$$

(4.13)

for third-order accuracy, and

$$\bar{q}^{i+1}_{i+1} = \frac{g^{i+1}_{i+1} + q^j}{2} + O(\Delta t^2) \text{ for } j = 0, 1, 2, \ldots$$

(4.14)

for second-order accuracy.

4.2.2 A numerical experiment

To test the accuracy of the CPMC method, we consider a synthetic channel flow

$$Q(x, t) = 2 + \sin \frac{2\pi x}{L} + \sin \frac{2\pi t}{T} \text{ (m}^3 \text{ s}^{-1})$$

(4.15)

for $0 \leq x \leq L$ and $0 \leq t \leq T$

with $L = 10,000$ m and $T = 10,000$ s. The width of the rectangular channel is 2 m, bed slope 0.01 so that the effect of the slope steepness on reference discharge can be neglected (Moramarco et al., 1999), and, Manning’s roughness coefficient 0.035.

The minimum inflow from Eq. (4.15), $Q_b = 1$ m$^3$ s$^{-1}$, and the peak inflow $Q_p = 3$ m$^3$ s$^{-1}$. We can calculate the reference discharge following Ponce and Chaganti (1994)

$$Q_b = \frac{Q_b + Q_p}{2} = 2 \text{ m}^3 \text{ s}^{-1}$$

(4.16)
We then obtain kinematic wave celerity \( C = 2.157 \text{ m s}^{-1} \), and diffusion coefficient \( D = 50.0 \text{ m}^2 \text{ s}^{-1} \). From Eq. (4.8), with the spatial interval \( \Delta x = L \), the time step for the third-order accuracy CPMC is \( \Delta t = 4637 \text{ s} \), and the Courant number is \( C_r = \frac{C\Delta t}{\Delta x} = 1.00 \). But with this time step, there are only a few points within range of the simulation time, and much information on temporally and spatially variable discharge would be lost. For easy comparison of the CPMC with the analytical solution, we may choose different time-step sizes, e.g., 1, 2, 5, 10, 20, 50, 100, 200, 500, and 1000 s, and divide the channel into multiple segments \( (n_s) \). For the second-order accuracy CPMC, we need to keep \( C_r \) as close to 1 as possible in our spatial discretization for any specific time-step size (Ponce, 1995, p. 294). For the third-order accuracy CPMC, we divide the channel into multiple segments following Eq. (4.7). If the channel length is not dividable by the required spatial interval for the third-order accuracy, we would make it as close as practical, and in this case the accuracy would be slightly lower than third order.

From Eq. (4.15), we have

\[
Q_t = \frac{2\pi}{T} \cos \frac{2\pi}{T} \\
Q_x = \frac{2\pi}{L} \cos \frac{2\pi x}{L} \\
Q_{xx} = \left( \frac{2\pi}{L} \right)^2 \sin \frac{2\pi x}{L}
\]  

(4.17)  

(4.18)  

(4.19)

Incorporating Eqs. (4.17)–(4.19) into (4.1) and simplifying, we obtain the lateral inflow

\[
q(x, t) = \frac{2\pi}{CT} \cos \frac{2\pi}{T} + \frac{2\pi}{L} \cos \frac{2\pi}{L} + D \left( \frac{2\pi}{L} \right)^2 \sin \frac{2\pi}{L}
\]  

(4.20)
So our channel routing problem is composed of Eq. (4.1), initial condition $Q(x,0) = 2 + \sin \frac{2\pi x}{L}$, boundary condition $Q(0,t) = 2 + \sin \frac{2\pi}{T}$, and lateral inflow calculated by Eq. (4.20). The channel routing results from second- and third-order accuracy CPMC method are compared with the analytical solution calculated by Eq. (4.15) at $x=L$. The results of CPMC method with average lateral inflow calculated by Eq. (4.14) are also compared with the analytical solution. Since the lateral inflow $q(x,t)$ defined by Eq. (4.20) is not uniformly distributed, we name the method of calculating $\bar{q}_{x}^{i+1}$ by Eq. (4.14) as the simplified method. In the simplified method, we still use the actual values of lateral inflow that are variable with space and time, but the spatial derivatives of lateral inflow are neglected. For uniformly distributed lateral inflow, this simplified method recovers the second-order accuracy.

To calculate $\bar{q}_{x}^{i+1}$ in the CPMC using Eq. (4.11) or (4.12), we also need the following derivatives of $q(x,t)$

$$q_{t} = -\frac{1}{C} \left( \frac{2\pi}{T} \right)^{2} \sin \frac{2\pi}{T}$$  \hspace{1cm}  \text{(4.21)}

$$q_{xx} = -\frac{1}{C} \left( \frac{2\pi}{T} \right)^{3} \cos \frac{2\pi}{T}$$  \hspace{1cm}  \text{(4.22)}

$$q_{x} = -\left( \frac{2\pi}{L} \right)^{2} \sin \frac{2\pi x}{L} + \frac{D}{C} \left( \frac{2\pi}{L} \right)^{3} \cos \frac{2\pi x}{L}$$  \hspace{1cm}  \text{(4.23)}

$$q_{xx} = -\frac{2\pi}{L} \cos \frac{2\pi x}{L} - \frac{D}{C} \left( \frac{2\pi}{L} \right)^{4} \sin \frac{2\pi x}{L}$$  \hspace{1cm}  \text{(4.24)}

$$q_{3x} = \left( \frac{2\pi}{L} \right)^{4} \sin \frac{2\pi x}{L} - \frac{D}{C} \left( \frac{2\pi}{L} \right)^{5} \cos \frac{2\pi x}{L}$$  \hspace{1cm}  \text{(4.25)}
4.3 Results

The simulated time to peak \( (t_p) \) by the second- and third-order CPMC methods compare well with the analytical solution of 2500 s for \( \Delta t \leq 100 \) s (Tables 4.1 and 4.2). For \( \Delta t = 200, 500, \) and 1000 s, the second-order CPMC resulted in smaller \( t_p \). The third-order CPMC led to smaller \( t_p \) for \( \Delta t = 200 \) and 1000 s. Both methods adequately estimated the peak discharge \( (Q_p, 3 \text{ m}^3 \text{ s}^{-1}) \).

The RMSE for the third-order CPMC solution is 2–18 times smaller than for the second-order CPMC for each corresponding \( \Delta t \) (Table 4.1 and 4.2). The RMSE for both methods decreases with \( \Delta t \) for \( \Delta t \geq 20 \) s, and remains nearly constant for \( \Delta t < 20 \) s (Fig. 4.2).

For large time steps, the spatial discretizations of second- and third-order accuracy scheme are the same (Tables 4.1 and 4.2). For small time steps, however, the third-order accuracy scheme need fewer spatial steps to reach an improved accuracy than the second-order accuracy scheme, even the third-order accuracy criterion \( C_r^2 + \frac{3}{P_e^2} - 1 = 0 \) or \( C_r = \sqrt{1 - \frac{3}{P_e^2}} \) is not fully met.

For the simplified method, the average lateral inflow are calculated by ignoring the spatial derivatives of lateral inflow but still accounting for different lateral inflow values at different locations. The simulated \( Q_p \) was underestimated for \( \Delta t \leq 20 \) s and overestimated for \( \Delta t \geq 50 \) s (Table 4.3). The simulated \( t_p \) was comparable with the third-order accuracy scheme for \( \Delta t \geq 20 \) s but over-estimated for \( \Delta t \leq 10 \) s.
The RMSE of the simplified method was generally larger than that of the second- or third-order scheme (Fig. 4.2). The RMSE was smallest when $\Delta t$ was close to 20 s, and increased with decreasing and increasing $\Delta t$. One exception was when $\Delta t=1000$ s, the results were more accurate than when $\Delta t=200$ or 500 s, and were comparable with the third-order accuracy solution. Hence, for this special example, the spatial derivatives of lateral inflow can be neglected if $\Delta t$ and $\Delta x$ were set as 1000 s and 2000 m, respectively.

The largest errors for CPMC solutions of different order of accuracy occur at different times. The largest errors for the second-order CPMC method occur before and after the peak, being overestimates before, and underestimates after, the peak (Fig. 4.3). The largest error for the third-order or the simplified method occurs only around the peak. The simulation results by the second- and third-order CPMC methods matched the analytical solution well for $\Delta t<500$ s, and by the simplified method for $\Delta t<100$ s.

### 4.4 Conclusions

For constant-parameter Muskingum-Cunge diffusion-wave channel routing with spatially and temporally variable lateral inflow, the accuracy of lateral inflow calculation is an important factor affecting the overall channel routing accuracy. In this study, we derived the average lateral inflow term in the second- and third-order accuracy CPMC methods for channel routing. The derived equations indicated that for spatially and temporally variable lateral inflow, the effect of lateral inflow on simulated discharge depended not only on the value of lateral inflow, its spatial and temporal derivatives, the spatial and temporal discretizations, but also on wave celerity and diffusion coefficient of the channel flow.

The second-order CPMC method led to increased accuracy with decreasing time-step sizes, and kept relatively constant for further decreased time-step sizes. Using larger time-step sizes is
computationally more efficient, but with higher risk of missing the exact peak discharge point by as much as one time step.

The accuracy of the third-order CPMC solution increased with decreasing time-step sizes, and was higher than the second-order CPMC method, even when the third-order accuracy CPMC method requirement \( C_r^2 + \frac{3}{P_e^2} - 1 = 0 \) was not fully satisfied because of limitation of constant temporal and spatial intervals used. Its computational costs can be much lower than the second-order CPMC method for smaller time-step sizes when it required few spatial steps. For larger time steps, its spatial discretization became the same as for the second-order scheme. This suggested that for a fixed time step, we can get second-order accuracy CPMC method by maintaining a Courant number of as close to 1 as practical, and obtain a higher accuracy by using a larger spatial step or a smaller Courant number, \( C_r = \sqrt{1 - \frac{3}{P_e^2}} \), with the condition that \( P_e \geq \sqrt{3} \).

When we ignore the spatial derivatives of the lateral inflow as in the simplified method, the RMSE of the numerical channel routing results was generally larger than that of the second- and third-order accuracy schemes. It was smallest for a time step of 20 s, and increased with both decreasing and increasing of the time-step size. Only for a special discretization, the simplified method led to the same result with the third-order accuracy scheme.

The second-order accuracy CPMC led to overestimation before and underestimation after, the time of peak discharge. The third-order accuracy CPMC and the simplified method only led to over- or underestimation near the time of peak discharge.
4.5 Tables and Figures
Table 4.1. Accuracy of the second-order CPMC channel routing with lateral inflow for different time-step sizes.

<table>
<thead>
<tr>
<th>$\Delta t$, s</th>
<th>$n_s$</th>
<th>$\Delta x$, m</th>
<th>RMSE, m$^3$ s$^{-1}$</th>
<th>$Q_p$, m$^3$ s$^{-1}$</th>
<th>$t_p$, s</th>
<th>$C_r$</th>
<th>$\Delta Q_p$, m$^3$ s$^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4637</td>
<td>2.16</td>
<td>9.82E-05</td>
<td>2.99995</td>
<td>2499</td>
<td>1.000</td>
<td>-4.90E-05</td>
</tr>
<tr>
<td>2</td>
<td>2318</td>
<td>4.31</td>
<td>9.86E-05</td>
<td>2.99995</td>
<td>2500</td>
<td>1.000</td>
<td>-4.91E-05</td>
</tr>
<tr>
<td>5</td>
<td>927</td>
<td>10.79</td>
<td>1.02E-04</td>
<td>2.99995</td>
<td>2500</td>
<td>1.000</td>
<td>-4.92E-05</td>
</tr>
<tr>
<td>10</td>
<td>464</td>
<td>21.55</td>
<td>1.12E-04</td>
<td>2.99995</td>
<td>2500</td>
<td>1.001</td>
<td>-5.25E-05</td>
</tr>
<tr>
<td>20</td>
<td>232</td>
<td>43.10</td>
<td>1.56E-04</td>
<td>2.99995</td>
<td>2500</td>
<td>1.001</td>
<td>-5.25E-05</td>
</tr>
<tr>
<td>50</td>
<td>93</td>
<td>107.53</td>
<td>4.77E-04</td>
<td>2.99992</td>
<td>2500</td>
<td>1.003</td>
<td>-8.34E-05</td>
</tr>
<tr>
<td>100</td>
<td>46</td>
<td>217.39</td>
<td>1.66E-03</td>
<td>2.99969</td>
<td>2500</td>
<td>0.992</td>
<td>-3.07E-04</td>
</tr>
<tr>
<td>200</td>
<td>23</td>
<td>434.78</td>
<td>6.32E-03</td>
<td>2.99774</td>
<td>2400</td>
<td>0.992</td>
<td>-2.26E-03</td>
</tr>
<tr>
<td>500</td>
<td>9</td>
<td>1111.11</td>
<td>3.99E-02</td>
<td>2.97296</td>
<td>2000</td>
<td>0.970</td>
<td>-2.70E-02</td>
</tr>
<tr>
<td>1000</td>
<td>5</td>
<td>2000.00</td>
<td>1.39E-01</td>
<td>2.98038</td>
<td>2000</td>
<td>1.078</td>
<td>-1.96E-02</td>
</tr>
</tbody>
</table>
Table 4.2. Accuracy of the third-order CPMC channel routing with lateral inflow for different time-step sizes.

<table>
<thead>
<tr>
<th>$\Delta t$, s</th>
<th>$n_s$</th>
<th>$\Delta x$, m</th>
<th>RMSE, m$^3$ s$^{-1}$</th>
<th>$Q_{pv}$, m$^3$ s$^{-1}$</th>
<th>$t_p$, s</th>
<th>$C_r$</th>
<th>$\Delta Q_{pv}$, m$^3$ s$^{-1}$</th>
<th>$C_r^2 + \frac{3}{P_r^2} - 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>124</td>
<td>80.65</td>
<td>1.63E-05</td>
<td>3.00004</td>
<td>2500</td>
<td>0.027</td>
<td>3.83E-05</td>
<td>7.52E-03</td>
</tr>
<tr>
<td>2</td>
<td>124</td>
<td>80.65</td>
<td>1.59E-05</td>
<td>3.00004</td>
<td>2500</td>
<td>0.053</td>
<td>3.75E-05</td>
<td>5.37E-03</td>
</tr>
<tr>
<td>5</td>
<td>123</td>
<td>81.30</td>
<td>1.52E-05</td>
<td>3.00004</td>
<td>2500</td>
<td>0.133</td>
<td>3.59E-05</td>
<td>6.57E-03</td>
</tr>
<tr>
<td>10</td>
<td>120</td>
<td>83.33</td>
<td>1.44E-05</td>
<td>3.00003</td>
<td>2500</td>
<td>0.259</td>
<td>3.38E-05</td>
<td>4.21E-03</td>
</tr>
<tr>
<td>20</td>
<td>110</td>
<td>90.91</td>
<td>1.41E-05</td>
<td>3.00003</td>
<td>2500</td>
<td>0.474</td>
<td>3.25E-05</td>
<td>5.57E-03</td>
</tr>
<tr>
<td>50</td>
<td>74</td>
<td>135.14</td>
<td>2.65E-05</td>
<td>3.00006</td>
<td>2500</td>
<td>0.798</td>
<td>6.32E-05</td>
<td>1.01E-02</td>
</tr>
<tr>
<td>100</td>
<td>43</td>
<td>232.56</td>
<td>1.10E-04</td>
<td>3.00027</td>
<td>2500</td>
<td>0.927</td>
<td>2.65E-04</td>
<td>2.08E-02</td>
</tr>
<tr>
<td>200</td>
<td>23</td>
<td>434.78</td>
<td>6.64E-04</td>
<td>2.99956</td>
<td>2400</td>
<td>0.992</td>
<td>-4.40E-04</td>
<td>1.83E-02</td>
</tr>
<tr>
<td>500</td>
<td>9</td>
<td>1111.11</td>
<td>1.17E-02</td>
<td>3.02719</td>
<td>2500</td>
<td>0.970</td>
<td>2.72E-02</td>
<td>5.29E-02</td>
</tr>
<tr>
<td>1000</td>
<td>5</td>
<td>2000.00</td>
<td>5.94E-02</td>
<td>3.09185</td>
<td>2000</td>
<td>1.078</td>
<td>9.19E-02</td>
<td>1.64E-01</td>
</tr>
</tbody>
</table>
Table 4.3. Accuracy of the CPMC channel routing with simplified calculation of lateral inflow (assuming uniformly distributed) for different time-step sizes.

<table>
<thead>
<tr>
<th>$\Delta t$, s</th>
<th>$n_s$</th>
<th>$\Delta x$, m</th>
<th>RMSE, m$^3$ s$^{-1}$</th>
<th>$Q_p$, m$^3$ s$^{-1}$</th>
<th>$t_p$, s</th>
<th>$C_r$</th>
<th>$\Delta Q_p$, m$^3$ s$^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4637</td>
<td>2.16</td>
<td>1.14E-02</td>
<td>2.97331</td>
<td>2512</td>
<td>1.000</td>
<td>-2.67E-02</td>
</tr>
<tr>
<td>2</td>
<td>2318</td>
<td>4.31</td>
<td>1.09E-02</td>
<td>2.97461</td>
<td>2512</td>
<td>1.000</td>
<td>-2.54E-02</td>
</tr>
<tr>
<td>5</td>
<td>927</td>
<td>10.79</td>
<td>9.19E-03</td>
<td>2.97851</td>
<td>2510</td>
<td>0.996</td>
<td>-2.15E-02</td>
</tr>
<tr>
<td>10</td>
<td>464</td>
<td>21.55</td>
<td>6.41E-03</td>
<td>2.98499</td>
<td>2510</td>
<td>1.001</td>
<td>-1.50E-02</td>
</tr>
<tr>
<td>20</td>
<td>232</td>
<td>43.10</td>
<td>8.51E-04</td>
<td>2.99798</td>
<td>2500</td>
<td>1.001</td>
<td>-2.02E-03</td>
</tr>
<tr>
<td>50</td>
<td>93</td>
<td>107.53</td>
<td>1.58E-02</td>
<td>3.03686</td>
<td>2500</td>
<td>1.003</td>
<td>3.69E-02</td>
</tr>
<tr>
<td>100</td>
<td>46</td>
<td>217.39</td>
<td>4.40E-02</td>
<td>3.10319</td>
<td>2500</td>
<td>0.992</td>
<td>1.03E-01</td>
</tr>
<tr>
<td>200</td>
<td>23</td>
<td>434.78</td>
<td>9.97E-02</td>
<td>3.23506</td>
<td>2400</td>
<td>0.992</td>
<td>2.35E-01</td>
</tr>
<tr>
<td>500</td>
<td>9</td>
<td>1111.11</td>
<td>2.72E-01</td>
<td>3.64472</td>
<td>2500</td>
<td>0.970</td>
<td>6.45E-01</td>
</tr>
<tr>
<td>1000</td>
<td>5</td>
<td>2000.00</td>
<td>5.94E-02</td>
<td>3.09185</td>
<td>2000</td>
<td>1.078</td>
<td>9.19E-02</td>
</tr>
</tbody>
</table>
Figure 4.1. Schematic of the relationship between hillslopes and a channel segment in the Water Erosion Prediction Project (WEPP) model.
Figure 4.2. Root-mean-square errors (RMSE) of the simplified, second-, and third-order CPMC.
Figure 4.3. Analytical solution and differences in discharge between numerical and analytical solutions. (a) second-order accuracy CPMC, (b) third-order accuracy CPMC, and (c) simplified method. Note different scales used for difference in discharge in (a), (b), and (c).
CHAPTER 5
SENsitivitY ANALYSIS OF CHANNeL-ROUTING PARAMetERS IN CURRENT
CHANNeL ROUTING METHODS IN THE WEPP MODEL

There are five methods to calculate channel flow for erosion estimation in the WEPP model. The
two original methods, the modified rational method used in the Erosion Productivity Impact
Calculator (EPIC) model (Williams, 1990), and the regression equation used in the Chemicals,
Runoff, and Erosion from Agricultural Management Systems (CREAMS) model (Knisel, 1980),
can be used to calculate peak discharge (Flanagan and Livingston, 1995). The newly added
channel-routing methods (Appendix B) in WEPP (v2010.1) are the linear kinematic-wave (LKW)
(Chow et al., 1988), the constant-parameter Muskingum-Cunge (CPMC), and the modified three-
point variable-parameter Muskingum-Cunge (MVPMC3) methods (Ponce and Chaganti, 1994).

To run WEPP with the newly incorporated channel-routing routines, user needs to set the flag
for the runoff peak calculation method (parameter \(i_{peak}\)) in the watershed channel file to 3, 4, or
5, corresponding to channel-routing method of LKW, CPMC, or MVPMC3. The values 1 and 2
for \(i_{peak}\) are used for the modified EPIC and CREAMS methods, respectively, as in the original
WEPP.

To generate channel-routing outputs for \(i_{peak}>2\), user needs to create an input file chan.inp
before running WEPP. The input parameters are described in Table 5.1. An example is given in
Table 5.2.

In this chapter, I present a comparison of the sensitivity of the peak discharge to channel-
routing parameters by the five methods of the modified EPIC, CREAMS, LKW, CPMC, and
MVPMC3 using a hypothetical watershed with three hillslopes and one channel. The shape of
the channel cross-section is parabolic. The parameters tested include time-step size ($\Delta t$) for channel routing, channel bed slope ($S_0$), channel length ($L$), channel cross-sectional width ($B$), and channel bank inverse slope ($z$). The spatial interval for channel routing is calculated as a function of $\Delta t$ to maintain a Courant number as close to one as possible. As such, sensitivity analysis of the spatial interval can be done similarly and will not be included here.

### 5.1 Time-step size

The original methods for channel flow calculation, i.e., the modified EPIC and CREAMS, give only daily peak discharge, which is not affected by $\Delta t$ for channel routing. The simulated peak discharge by LKW, CPMC, or MVPMC3, however, was affected by $\Delta t$ as expected (Table 5.3). Compared with the peak discharge using $\Delta t = 60$ s, the simulated peak discharge using $\Delta t = 300$, 600, or 1200 s was attenuated by 11%, 14%, or 26%, respectively, for LKW method, and by 2%, 6%, or 14%, respectively, for the CPMC or MVPMC3 method. The attenuation caused by the CPMC or MVPMC3 was smaller than by the LKW method.

The resultant wave attenuation was caused not only by dissipation of the numerical channel routing methods (Chow et al., 1988; Wang et al., 2010), but also by missing the points of peak flow of the upstream inflow and lateral inflow data (Fig. 5.1). For the hypothetical case, the simulation results using a $\Delta t$ of 300 s is almost the same as using a $\Delta t$ of 60 s. But a $\Delta t$ of 1200 s will result in a much larger error. Generally, if the rising and falling limbs of the runoff hydrographs generated from all the hillslopes are relatively gentle, using a larger $\Delta t$ may not cause substantial increase in error by missing the points near or at the peak; however, if there are waves with steep rising or falling limbs, the attenuation caused by using a large $\Delta t$ may be significant. Hence, a small $\Delta t$ should be used whenever possible.
5.2 Channel bed slope

Channel bed slope affected the simulated peak discharge in all methods (Table 5.4). The effect on the simulated peak discharge by the CREAMS method was the most substantial, followed by that from the EPIC method. With an increase of $S_0$ from 0.03 to 0.3, the simulated peak discharge using LKW, CPMC, and MVPMC3 will increase by 4–5%. When $S_0$ is decreased from 0.03 to 0.003, the simulated peak discharge is decreased by 11% using LKW, and by 6% using the CPMC or MVPMC3.

5.3 Channel length

The simulated peak discharge was most affected by channel length using the EPIC method, and only slightly when using the CREAMS method (Table 5.5). The simulated peak discharge from LKW is lower than from CPMC and MVPMC3. For longer channels, the effect on the simulated peak flow is most evident using LKW (Fig. 5.2). The difference between the CPMC and MVPMC3 results is that a wave moves at a constant speed in the former and at variable speed in the latter, thus leading to the steepening of the wave front. Wave front steepening can also be simulated using LKW method.

5.4 Channel width

For all methods, the effect of channel width on the simulated peak discharge was inconsequential, less than 2% when channel width was decreased from 1 m to 0.1 m, or increased from 1 m to 10 m (Table 5.6). Therefore, peak flow is not sensitive to channel width in any of methods for channel flow calculation in the current WEPP.

5.5 Channel inverse slope

The simulated peak discharge by EPIC and CREAMS method remained unchanged when the inverse slope of channel bank was varied from 2 to 200 (Table 5.7). The simulated peak
discharge by the LKW, CPMC, or MVPMC3 method was changed by less than 1% when inverse slope was decreased from 20 to 2; and less than 2% when the inverse slope was increased from 20 to 200. Consequently, the simulated peak discharge is not sensitive to inverse slope of channel bank using any method for channel flow in the current WEPP.

5.6 Summary

The LKW, CPMC, and MVPMC3 methods can be used in WEPP channel routing to obtain hydrographs for all the channels in a watershed. Among the three methods, LKW is the most stable, but can lead to numerical dissipation (Table 5.8). It is suitable for a small watershed with relatively short and steep channels. To prevent numerical dissipation, user should use small time-step sizes.

The results from the CPMC and MVPMC3 methods are generally similar, with less numerical dissipation compared to the LKW method. For long channels, the steepening of the wave front can be better simulated using the MVPMC3 method. The disadvantage of the CPMC and MVPMC3 methods is the potential numerical dispersion, which, however, can be decreased or eliminated by using smaller time-step sizes.
5.7 Tables and Figures
Table 5.1. Input file *chan.inp* for channel routing in WEPP (later than v2010.1).

<table>
<thead>
<tr>
<th>Line</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Line 1:</strong></td>
<td>time step (<em>dtchr</em>, s), and output option (<em>ichout</em>)</td>
</tr>
<tr>
<td><em>dtchr:</em></td>
<td>60–3600 s</td>
</tr>
<tr>
<td><em>ichan:</em></td>
<td>( \leq 0 ) - no channel-routing output (default)</td>
</tr>
<tr>
<td>1</td>
<td>output peak flow time and rate (<em>chan.out</em>) for, and channel water balance (<em>chanwb.out</em>) above, specified channels</td>
</tr>
<tr>
<td>2</td>
<td>output daily average flow rate and total runoff (<em>chan.out</em>) for, and channel water balance above, specified channels</td>
</tr>
<tr>
<td>3</td>
<td>output flow rate (<em>chan.out</em>) at each time step for, and channel water balance above, specified channels.</td>
</tr>
<tr>
<td><strong>Line 2:</strong></td>
<td>unit-area baseflow coefficient (<em>cbase</em>, ( \text{m}^3 \text{s}^{-1} \text{m}^{-2} ))</td>
</tr>
<tr>
<td><strong>Line 3:</strong></td>
<td>number of channels for writing channel-routing output - integer (<em>nchnum</em>)</td>
</tr>
<tr>
<td>( \leq 0 )</td>
<td>no channel-routing output (default)</td>
</tr>
<tr>
<td><strong>Line 4:</strong></td>
<td>channel element IDs for writing output - integer (<em>ichnum</em>)</td>
</tr>
<tr>
<td>If not specified</td>
<td>no channel-routing output (default).</td>
</tr>
</tbody>
</table>
Table 5.2. An example of channel input file (chan.inp).

300.0, 3 ! time step (s), output option
0.0E+0 ! Unit-area baseflow coefficient
1 ! number of channels for writing channel routing output
123 ! channel element ID for writing output
Table 5.3. Sensitivity of simulated peak discharge to time-step size.

<table>
<thead>
<tr>
<th>Time-step size (s)</th>
<th>EPIC</th>
<th>CREAMS</th>
<th>LKW</th>
<th>CPMC</th>
<th>MVPMC3</th>
</tr>
</thead>
<tbody>
<tr>
<td>600</td>
<td>0.247</td>
<td>0.663</td>
<td>0.136</td>
<td>0.152</td>
<td>0.153</td>
</tr>
<tr>
<td>300</td>
<td>0.247</td>
<td>0.663</td>
<td>0.141</td>
<td>0.158</td>
<td>0.158</td>
</tr>
<tr>
<td>60</td>
<td>0.247</td>
<td>0.663</td>
<td>0.159</td>
<td>0.161</td>
<td>0.163</td>
</tr>
<tr>
<td>1200</td>
<td>0.247</td>
<td>0.663</td>
<td>0.117</td>
<td>0.139</td>
<td>0.139</td>
</tr>
</tbody>
</table>
Table 5.4. Sensitivity of simulated peak discharge to channel bed slope.

<table>
<thead>
<tr>
<th>Bed slope</th>
<th>EPIC</th>
<th>CREAMS</th>
<th>LKW</th>
<th>CPMC</th>
<th>MVPMC3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.03</td>
<td>0.247</td>
<td>0.663</td>
<td>0.136</td>
<td>0.152</td>
<td>0.153</td>
</tr>
<tr>
<td>0.3</td>
<td>0.281</td>
<td>0.956</td>
<td>0.142</td>
<td>0.159</td>
<td>0.160</td>
</tr>
<tr>
<td>0.003</td>
<td>0.192</td>
<td>0.460</td>
<td>0.121</td>
<td>0.142</td>
<td>0.143</td>
</tr>
</tbody>
</table>
Table 5.5. Sensitivity of simulated peak discharge to channel length.

<table>
<thead>
<tr>
<th>Length (m)</th>
<th>EPIC</th>
<th>CREAMS</th>
<th>LKW</th>
<th>CPMC</th>
<th>MVPMC3</th>
</tr>
</thead>
<tbody>
<tr>
<td>120</td>
<td>0.247</td>
<td>0.663</td>
<td>0.136</td>
<td>0.152</td>
<td>0.153</td>
</tr>
<tr>
<td>12</td>
<td>0.304</td>
<td>0.662</td>
<td>0.145</td>
<td>0.164</td>
<td>0.164</td>
</tr>
<tr>
<td>1200</td>
<td>0.107</td>
<td>0.671</td>
<td>0.101</td>
<td>0.142</td>
<td>0.153</td>
</tr>
</tbody>
</table>
Table 5.6. Sensitivity of simulated peak discharge to channel width.

<table>
<thead>
<tr>
<th>Width (m)</th>
<th>Peak discharge (m$^3$ s$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>EPIC</td>
</tr>
<tr>
<td>1</td>
<td>0.247</td>
</tr>
<tr>
<td>0.1</td>
<td>0.247</td>
</tr>
<tr>
<td>10</td>
<td>0.251</td>
</tr>
</tbody>
</table>
Table 5.7. Sensitivity of simulated peak discharge to channel inverse slope.

<table>
<thead>
<tr>
<th>Inverse slope</th>
<th>Peak discharge (m³ s⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>EPIC</td>
</tr>
<tr>
<td>20</td>
<td>0.247</td>
</tr>
<tr>
<td>2</td>
<td>0.247</td>
</tr>
<tr>
<td>200</td>
<td>0.247</td>
</tr>
</tbody>
</table>
Table 5.8. A guide on selecting channel routing methods.

<table>
<thead>
<tr>
<th>Method</th>
<th>Linear kinematic wave</th>
<th>Constant-parameter Muskingum-Cunge</th>
<th>Modified variable-parameter Muskingum-Cunge</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wave type</td>
<td>Kinematic</td>
<td>Diffusion</td>
<td>Diffusion</td>
</tr>
<tr>
<td>Dissipation</td>
<td>Yes</td>
<td>Negligible</td>
<td>Negligible</td>
</tr>
<tr>
<td>Dispersion</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Steepening</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Note</td>
<td>Use smaller $\Delta t$ to reduce numerical dissipation</td>
<td>Use smaller $\Delta t$ to minimize dispersion</td>
<td>Use smaller $\Delta t$ to minimize dispersion</td>
</tr>
<tr>
<td>Applicable</td>
<td>Small- to medium-size watersheds with steep and short channels</td>
<td>Small to large watersheds</td>
<td>Small to large watersheds</td>
</tr>
</tbody>
</table>
Figure 5.1. Effects of time-step size on input data selection for channel routing. (a) upstream channel inflow, (b) lateral channel inflow. Note the results for 60 s and 300 s largely overlap.
Figure 5.2. Effects of channel length on simulated discharge. (a) Linear kinematic-wave (LKW), (b) constant-parameter Muskingum-Cunge (CPMC), and (c) modified three-point variable-parameter Muskingum-Cunge (MVP-MC3).
CHAPTER 6
SUMMARY AND CONCLUSIONS

Channel routing is an important component in watershed modeling. This dissertation examines common channel-routing methods and their applications to commonly used watershed models, such as the Water Erosion Prediction Project (WEPP) model. The primary goal of this study was to investigate different channel-routing methods that are suitable for watershed modeling and to develop the channel-routing routines and incorporate them into the selected watershed model, the WEPP model.

To accomplish this goal, I examined kinematic- and diffusion-wave models and incorporated the kinematic-wave and the Muskingum-Cunge methods into WEPP. I then applied these methods to a hypothetical channel system, and to North Fork Caspar Creek Watershed, CA. These case applications showed that the kinematic-wave method was robust and stable even for nonlinear waves, but the resultant numerical dissipation increased with the sizes of temporal and spatial discretization. The Muskingum-Cunge method was computationally more efficient, and the resultant peak attenuation was smaller, although numerical dispersion may occur if the temporal or spatial interval is inadequate.

The Hayami convolution solution can be used in constant-parameter diffusion-wave channel routing with distributed lateral inflow. However, the convolution of channel inflow with the continuous Hayami kernel function has analytical solutions only for simplified cases and is costly when solved using numerical integration for watershed modeling that often involves hundreds of reaches. It is generally more efficient to use the discrete Hayami convolution, with the time-step size small enough so that the discrete exact point values can be used in replacement
of the continuous Hayami kernel function. For our case applications, when the rising limb of the Hayami kernel function is shorter than 1.8 time steps in duration, the integration of the discrete Hayami kernel function using the point kernel function values is hardly unity, which would lead to mass balance error for channel routing. Hence, this solution applies only to cases with rising limb of the kernel function longer than 1.8 time steps. Not meeting this criterion can lead to considerable error in simulated mass balance. To avoid such error, one approach is to use the center-averaged kernel function values instead of the exact values. The integration of the discrete kernel function using the averaged kernel function values is always unity, regardless of temporal resolution.

For diffusion-wave channel routing with spatially and temporally variable lateral inflow, the accuracy of lateral inflow calculation is one of the major factors affecting the overall channel routing accuracy. In Chapter 4, the average lateral inflow term was derived for the second- and third-order accuracy constant-parameter Muskingum-Cunge (CPMC) channel routing methods. The derived equations indicate that for spatially and temporally variable lateral inflow, the effect of lateral inflow on simulated discharge would depend not only on the value of lateral inflow, its spatial and temporal derivatives, the spatial and temporal discretizations, but also on wave celerity and diffusion coefficient of the channel flow.

Linear kinematic-wave (LKW), CPMC, and the modified three-point variable-parameter Muskingum-Cunge (MVPMC3) for channel routing have been incorporated into the WEPP model. With these newly added channel routing routines, simulated channel hydrographs can be generated. A sensitivity analysis for a hypothetical watershed showed that the simulated peak discharge was sensitive to time-step size, channel length, and channel bed slope, and not sensitive to channel width and channel bank inverse slope.
For future work, the discrete Hayami convolution method and the second- and third-order accuracy CPMC methods may be implemented in the WEPP model to further enhance its general applicability.
REFERENCES


Wang, G.-T., Yao, C., Okoren, C., Chen S., 2006. 4-Point FDF of Muskingum method based on the complete St Venant equations, J. Hydrol. 324, 339–349.


APPENDIX

A. Derivation of the third-order accuracy CPMC method for constant-parameter diffusion-wave channel routing with lateral inflow

In order to obtain the 3rd-order accurate solution of Eq. (4.2), we calculate the derivatives of $Q$ with respect to space and time, and represent them as the derivatives of space only.

First, we rearrange Eq. (4.1) as

$$Q_t = -CQ_x + DQ_{xx} + Cq$$

(A1)

where,

$$Q_t = \frac{\partial Q}{\partial t}(x,t), \quad Q_x = \frac{\partial Q}{\partial x}(x,t), \quad Q_{xx} = \frac{\partial^2 Q}{\partial x^2}(x,t), \text{ and } q = q(x,t)$$

are used for brevity, and similar notations are used for the following derivations.

The derivatives are then

$$Q_{xx} = Q_{xx} = -CQ_{xx} + DQ_{x} + Cq_x$$

(A2)

$$Q_{xxx} = (Q_{xx})_x = -CQ_{3x} + DQ_{4x} + Cq_{3x}$$

(A3)

$$Q_{xx} = -CQ_{3x} + DQ_{x} + Cq_{3x}$$

$$Q_{xxx} = (Q_{xx})_x = -CQ_{3x} + DQ_{4x} + Cq_{3x}$$

(A4)

$$Q_{xxx} = (Q_{xx})_x = -CQ_{4x} + DQ_{5x} + Cq_{3x}$$

(A5)

$$Q_{x} = (Q_{xx})_x = -CQ_{x} + DQ_{3x} + Cq_{x}$$

$$Q_{xxx} = (Q_{xx})_x = -CQ_{3x} + DQ_{4x} + Cq_{xx}$$

(A6)

$$Q_{xxx} = (Q_{xx})_x = -CQ_{3x} + DQ_{5x} + Cq_{xx}$$

(A7)
\[ Q_{jt} = (Q_{jt}) = (C^2 Q_{xx} - 2CDQ_{3xt} + D^2 Q_{4xt} + Cq_r - C^2 q_x + CDq_{xx})t, \]
\[ = C^2 Q_{xx} - 2CDQ_{3xt} + D^2 Q_{4xt} + Cq_r - C^2 q_x + CDq_{xx} = C^2(-CQ_{3x} + DQ_{4x} + Cq_{xx}) - 2CD(-CQ_{4x} + DQ_{5x} + Cq_{3x}) \]
\[ + D^2(-CQ_{5x} + DQ_{6x} + Cq_{4x}) + Cq_r - C^2 q_x + CDq_{xx} = -C^3Q_{3x} + C^2 DQ_{4x} + 2C^2 DQ_{4x} - 2CD^2 Q_{5x} - CD^2 Q_{5x} + D^3 Q_{6x} + C^3 q_{xx} \]
\[ - 2C^2 Dq_{3x} + CD^2 q_{4x} + Cq_r - C^2 q_x + CDq_{xx} = -C^3Q_{3x} + 3C^2 DQ_{4x} - 3CD^2 Q_{5x} + D^3 Q_{6x} + C^3 q_{xx} + Cq_r - C^2 q_x \]
\[ - 2C^2 Dq_{3x} + CDq_{xx} + CD^2 q_{4x} \]

Expand the term \( Q_{t+1}^j = Q(x + \Delta x, t + \Delta t) \) in Eq. (4.2) respect to \((x, t)\) to the third order Taylor series, neglect the superscript \( j \) in the derivatives at \((x, t)\) for brevity, and drop the higher order terms,

\[ Q_{t+1}^j = Q_j^t + Q_x \Delta x + Q_t \Delta t + \frac{1}{2} Q_{xx} \Delta x^2 + \frac{1}{2} Q_{tt} \Delta t^2 + Q_{xt} \Delta x \Delta t + \frac{1}{6} Q_{xxx} \Delta x^3 + \frac{1}{6} Q_{ttx} \Delta t^3 + \frac{1}{2} Q_{xxt} \Delta x^2 \Delta t + \frac{1}{2} Q_{xtx} \Delta x \Delta t^2 \]

Incorporating the derivative terms and rearranging gives

\[ Q_{t+1}^j = Q_j^t + Q_x (\Delta x - C \Delta t) + Q_{x} \left(D \Delta t + \frac{1}{2} \Delta x^2 + \frac{1}{2} C^2 \Delta t^2 - C \Delta x \Delta t \right) \]
\[ + Q_{3x} \left(-CD \Delta t^2 + D \Delta x \Delta t + \frac{1}{6} \Delta x^3 - \frac{1}{6} C^3 \Delta t^3 - \frac{1}{2} C \Delta x^2 \Delta t + \frac{1}{2} C^2 \Delta x \Delta t^2 \right) \]
\[ + C \Delta \left[q_j^t + \frac{1}{2} \Delta t(q_i - C q_x + D q_{xx}) + \Delta x q_x + \frac{1}{6} \Delta t^2 \left(C^2 q_{xx} + q_r - C q_{xx} - 2CD q_{3x} \right) \right] + D q_{xxt} + D^2 q_{4x} \right) + \frac{1}{2} \Delta x^2 q_{xx} + \frac{1}{2} \Delta x \Delta t \left(q_r - C q_{xx} + D q_{3x} \right) \right] \]

Similarly,
\begin{align}
Q_{i+1}^j & = Q_i^j + Q_x(-C\Delta t) + Q_{xx}\left(D\Delta t + \frac{1}{2}C^2\Delta t^2\right) + Q_{3x}\left(-CD\Delta t^2 - \frac{1}{6}C^3\Delta t^3\right) \\
& \quad + C\Delta t\left[q_i^j + \frac{1}{2}\Delta t(q_i - Cq_x + Dq_{xx}) + \frac{1}{6}\Delta t^2\left(C^2q_{xx} + q_{tt} - Cq_{xt} - 2CDq_{3x} + Dq_{xx} + D^2q_{4x}\right)\right] \\
& \quad + C\Delta t\left[q_i^j + \frac{1}{2}\Delta t(q_i - Cq_x + Dq_{xx}) + \frac{1}{6}\Delta t^2\left(C^2q_{xx} + q_{tt} - Cq_{xt} - 2CDq_{3x} + Dq_{xx} + D^2q_{4x}\right)\right] \\
& = C_1Q_i^j \\
& \quad + C_2\left[Q_i^j + Q_x(-C\Delta t) + Q_{xx}\left(D\Delta t + \frac{1}{2}C^2\Delta t^2\right) + Q_{3x}\left(-CD\Delta t^2 - \frac{1}{6}C^3\Delta t^3\right)\right] \\
& \quad + C\Delta t\left[q_i^j + \frac{1}{2}\Delta t(q_i - Cq_x + Dq_{xx}) + \frac{1}{6}\Delta t^2\left(C^2q_{xx} + q_{tt} - Cq_{xt} - 2CDq_{3x} + Dq_{xx} + D^2q_{4x}\right)\right] \\
& \quad + C_3\left[Q_i^j + Q_x(\Delta x) + Q_{xx}\left(\frac{1}{2}\Delta x^2\right) + Q_{3x}\left(\frac{1}{6}\Delta x^3\right)\right] + C_4\bar{q}_{i+1}^j\Delta x \\
& = (A11)
\end{align}

Incorporating Eq. (A10), (A11), and (A12) into (4.2), we have
\begin{align}
Q_i^j & = Q_i^j + Q_x(\Delta x) + Q_{xx}\left(\frac{1}{2}\Delta x^2\right) + Q_{3x}\left(\frac{1}{6}\Delta x^3\right) \\
& = (A12)
\end{align}

Equate the coefficient terms related to $Q_i^j$, $Q_x$, $Q_{xx}$, $Q_{3x}$, and $q$, respectively,
\begin{align}
Q_i^j & : \quad 1 = C_1 + C_2 + C_3 \\
& = (A14) \\
Q_x & : \quad \Delta x - C\Delta t = C_2(-C\Delta t) + C_3\Delta x \\
& = (A15) \\
Q_{xx} & : \quad D\Delta t + \frac{1}{2}\Delta x^2 + \frac{1}{2}C^2\Delta t^2 - C\Delta x \Delta t = C_2\left(D\Delta t + \frac{1}{2}C^2\Delta t^2\right) + C_3\left(\frac{1}{2}\Delta x^2\right) \\
& = (A16)
\end{align}
\[ Q_{3x} : - CD\Delta t^2 + D\Delta x\Delta t + \frac{1}{6} C\Delta^3 + \frac{1}{2} C\Delta^2 \Delta t + \frac{1}{2} C^2 \Delta x \Delta t^2 \]

\[ = C_1 \left( - CD\Delta t^2 - \frac{1}{6} C\Delta^3 \right) + C_2 \left( \frac{1}{6} \Delta^3 \right) \]  

(A17)

\[ q : C\Delta t \left[ q_i^t + \frac{1}{2} \Delta t (q_{i-1} - Cq_x + Dq_{xx}) + \Delta t q_x + \frac{1}{6} \Delta t^2 (C^2 q_{xx} + q_{tt} - Cq_{xt} - 2CDq_{3x} + Dq_{3xt} + D^2 q_{4x}) \right. \]

\[ + \frac{1}{2} \Delta x^2 q_{xx} + \frac{1}{2} \Delta x \Delta t (q_{xt} - Cq_{xx} + Dq_{3x}) \left. \right] \]

\[ = C_2 C\Delta t \left[ q_i^t + \frac{1}{2} \Delta t (q_{i-1} - Cq_x + Dq_{xx}) + \frac{1}{6} \Delta t^2 (C^2 q_{xx} + q_{tt} - Cq_{xt} - 2CDq_{3x} + Dq_{3xt} + D^2 q_{4x}) \right. \]

\[ + C_4 q_{j+1}^{t+1} \Delta x \]  

(A18)

Solving the system equations of (A14)–(A16) gives the Muskingum-Cunge coefficients

\[ C_1 = \frac{\Delta x + C\Delta t - \frac{2D}{C}}{\Delta x + C\Delta t + \frac{2D}{C}} \]  

(A19)

\[ C_2 = \frac{- \Delta x + C\Delta t + \frac{2D}{C}}{\Delta x + C\Delta t + \frac{2D}{C}} \]  

(A20)

and

\[ C_3 = \frac{\Delta x - C\Delta t + \frac{2D}{C}}{\Delta x + C\Delta t + \frac{2D}{C}} \]  

(A21)

The coefficients are the same as that given by Chow et al. (1988) and Ponce (1995). And it shows that the CPMC method without further restriction is second-order accurate.

Incorporating Eq. (A20) and (A21) into (A17) and simplifying, we have

\[ C^4 \Delta t^2 - C^2 \Delta x^2 + 12D^2 = 0 \]  

(A22)
solving for $\Delta x$, we have

$$\Delta x = \sqrt{C^2 \Delta t^2 + \frac{12D^2}{C^2}} \quad (A23)$$

or solving for $\Delta t$, we have

$$\Delta t = \frac{1}{C} \sqrt{\Delta x^2 - \frac{12D^2}{C^2}} \quad (A24)$$

Eqs. (A23) and (A24) are the relationships between $\Delta x$ and $\Delta t$ required to maintain the third-order accurate for the CPMC method, and it has been derived by Bajracharya and Barry (1997) and Szel and Gaspar (2000) for CPMC method solving the diffusion-wave channel routing without lateral inflow.

Dividing both sides of (A22) by $C^2 \Delta x^2$ and introducing $C_r$ and $P_e$, we can simplify (A22) to a dimensionless equation required for eliminating the dispersion error to obtain the third-order CPMC method (Szel and Gaspar, 2000): $C_r^2 + \frac{3}{P_e^2} - 1 = 0$.

From Eq. (A24), in order for $\Delta t$ to be real, we must have

$$\Delta x > \frac{2\sqrt{3D}}{C} \quad (A25)$$

From Eq. (A18), we obtain

$$\begin{align*} C_x q_{x,i+1} \Delta x &= \frac{2C \Delta t \Delta x}{\Delta x + C \Delta t + \frac{2D}{C}} \left[ q_{i+1} + \frac{1}{2} q_{i} \Delta t + \frac{1}{2} q_{i+1} \left( \Delta x + \frac{2D}{C} \right) + \frac{1}{6} q_{i} \Delta t^2 + \frac{1}{4} q_{i+1} \left( \Delta x^2 - \frac{1}{3} C^2 \Delta t^2 + \frac{2D \Delta x}{C} \right) \right] \\
&+ \frac{1}{4} q_{x,i} \left( \Delta x + \frac{1}{3} C \Delta t + \frac{2D}{C} \right) \Delta t + \frac{1}{4} q_{3x} \left( \Delta x - \frac{1}{3} C \Delta t + \frac{2D}{C} \right) D \Delta t + \frac{1}{6} q_{xt} \Delta t^2 + \frac{1}{6} q_{4x} D^2 \Delta t^2 \end{align*} \quad (A26)$$

Letting
\[ C_4 = \frac{2C\Delta t}{\Delta x + C\Delta t + \frac{2D}{C}} \]  (A27)

we get

\[
\bar{q}_{i+1}^{j+1} = q_i^j + \frac{1}{2} q_i \Delta t + \frac{1}{2} q_x \left( \Delta x + \frac{2D}{C} \right) + \frac{1}{6} q_x \Delta t^2 + \frac{1}{4} q_{xx} \left( \Delta x - \frac{1}{3} C \Delta t^2 + \frac{2D\Delta x}{C} \right) + \frac{1}{4} q_{x\tau} \Delta t + \frac{1}{4} q_{3x} \left( \Delta x - \frac{1}{3} C \Delta t + \frac{2D}{C} \right) \Delta t + \frac{1}{6} q_{xx\tau} D\Delta t^2 + \frac{1}{6} q_{4x} D^2 \Delta t^2
\]  (A28)

And incorporating Eq. (A20) into (A27) and simplifying results in

\[ C_4 = \frac{2C\Delta t}{\Delta x + C\Delta t + \frac{2D}{C}} = C_1 + C_2 = 1 - C_3 \]  (A29)

Eqs. (4.2), (A19)–(A21), (A29), and (A28) together form the third-order CPMC method with spatial or temporal limitations defined by Eq. (A23) and (A24), respectively.

If the spatial variation of lateral inflow is negligible so that the derivatives of \( q \) with respect to \( x \) vanish in Eq. (A28), the average lateral inflow can also be estimated simply from a discrete dataset. Since (Thomas, 1995)

\[
q_i = \begin{cases} 
- \frac{q_i^{j+2} + 4q_i^{j+1} - 3q_i^j}{2\Delta t} + O(\Delta t^2) & \text{for } j = 0 \\
\frac{q_i^{j+1} - q_i^{j-1}}{2\Delta t} + O(\Delta t^2) & \text{for } j = 1, 2, \ldots 
\end{cases}
\]  (A30)

and

\[
q_{xx} = \begin{cases} 
\frac{q_i^{j+2} - 2q_i^{j+1} + q_i^j}{\Delta t^2} + O(\Delta t) & \text{for } j = 0 \\
\frac{q_i^{j+1} - 2q_i^j + q_i^{j-1}}{\Delta t^2} + O(\Delta t^2) & \text{for } j = 1, 2, \ldots 
\end{cases}
\]  (A31)

Incorporating Eqs. (A30) and (A31) into (A28) and simplifying, we have
\[
\bar{q}_{i+1}^{j+1} = \begin{cases} 
- q_i^{j+2} + 8q_i^{j+1} + 5q_i^j & + \Delta t(\Delta) \\
\frac{12}{5}q_i^{j+1} + 8q_i^j - q_i^{j-1} & + O(\Delta t^3) 
\end{cases} \quad \text{for } j = 0, 1, 2, \ldots
\] (A32)

Following the same procedure, the second-order accuracy lateral inflow term can be obtained as

\[
\bar{q}_{i+1}^{j+1} = q_i^j + \frac{1}{2} q_i \Delta t + \frac{1}{2} q_i \left( \Delta x + \frac{2D}{C} \right) + \frac{1}{2} q_{ix} D \Delta t
\] (A33)

If we ignore the spatial derivatives, and use \( q_i = \frac{q_i^{j+1} - q_i^j}{\Delta t} + O(\Delta t) \), Eq. (A33) can be simplified to (Chow et al., 1988)

\[
\bar{q}_{i+1}^{j+1} = q_i^j + \frac{1}{2} q_i \Delta t + \frac{1}{2} q_i \left( \Delta x + \frac{2D}{C} \right) + \frac{1}{2} q_{ix} D \Delta t
\] (A34)

For kinematic wave channel routing, \( D = 0 \), Eq. (A33) is simplified to

\[
\bar{q}_{i+1}^{j+1} = q_i^j + \frac{1}{2} q_i \Delta t + \frac{1}{2} q_i \Delta x
\] (A35)

If we estimate the derivatives \( q_x \) and \( q_x \) respectively, by \( q_x = \frac{q_i^{j+1} - q_i^j}{\Delta x} + O(\Delta x) \) and \( q_x = \frac{q_i^j - q_i^{j+1}}{\Delta t} + O(\Delta t) \), Eq. (A35) becomes

\[
\bar{q}_{i+1}^{j+1} = \frac{q_i^{j+1} + q_i^{j+1}}{2}.
\] (A36)
B. Channel routing codes incorporated in WEPP

B.1. WSHCHR.FOR

subroutine wshchr

     + + + PURPOSE + + +
     SR WSHCHR routes channel flow using either kinematic wave method
     or Muskingum-Cunge method.

     Called from: SR WSHDRV, SR WSHPEK
     Author(s): L. Wang, S. Dun, J. Frankenberger
     Reference in User Guide:

     February 08, 2012

     + + + KEYWORDS + + +

     + + + PARAMETERS + + +

     include 'pmxelm.inc'
     include 'pmxhil.inc'
     include 'pmxpln.inc'
     include 'pmxpri.inc'
     include 'pmxslo.inc'
     include 'pmxtim.inc'
     include 'pmxseg.inc'
     include 'pmxcsit.inc'
     include 'pmxtis.bit'
     include 'pmxchr.inc'

     + + + ARGUMENT DECLARATIONS + + +

     + + + ARGUMENT DEFINITIONS + + +

     q1    - channel outflow rate, [m3/s]

     + + + COMMON BLOCKS + + +

     include 'cchpek.inc'
     include 'cdata1.inc'
     include 'cdata3.inc'
     include 'chvdril.inc'
     include 'cslope.inc'
     include 'cstore.inc'
     include 'cstruc.inc'
     include 'cstruct.inc'
     include 'cchvar.inc'
include 'cchpar.inc'
include 'cchrt.inc'
include 'cupdate.inc'
include 'cslpopt.inc'

++ LOCAL VARIABLES ++

real qchpk, qchavg, ckref, cq, tk, cx, qlavg, qref, qtmax,
1 qmax, qmaxi, qmin, ck, c0, c1, c2, c3, c4, chvol, bw, y, eps,
1 dtdx, chn11, chns0, chn0, chnz0, volin, ain, aout, asum,
1 cxmin, aavg, qin(0:mxtchr), qlat(0:mxtchr),
1 qs(0:mxseg,0:mxtchr), bal, qmin, qbasel, vbasel, volon,
1 vsb, vsbbs, qbasel, vbaset, ckw, bt, aref, ap, dqdy,
1 sslp, qavg, ti, qsb, qtotl, areat, areal

integer i, it, itt, itpk, ih, is, nseg, ic, ishp, nt0

++ LOCAL DEFINITIONS ++

bw - channel width, [m]
chn1 - channel length, [m]
ishp - shape of the channel
it - current time step number
qbasel - channel lateral inflow from groundwater base flow of the side hillslopes, [m3/s]
qbaset - channel inflow from groundwater base flow of the top hillslope, [m3/s]
qin - channel inflow rate, [m3/s]
qlat - channel lateral inflow
qs - channel flow rate, [m3/s]
vbasel - channel lateral inflow volume from groundwater base flow, [m3]
vbaset - channel inflow volume from groundwater base flow, [m3]
volin - channel inflow or channel lateral inflow volume, [m3]

++ SUBROUTINES CALLED ++

chrqin
mann

++ DATA INITIALIZATIONS ++

++ END SPECIFICATIONS ++

c eps = 1.e-8
c qs = 0.
c qmax = 0.
c qmin = 0.
c qtmax = 0.
c qtmin = 0.
c volin = 0.
chnl1 = chnlen(ichan)
bw = chnwid(ichan)
ishp = ishape(ichan)
chns0 = chnslp(ichan,1)
chnn0 = chnn(ichan)
chnz0 = chnz(ichan)
if(chnz0 < eps) ishp = 2
if(ishp == 3) chnz0 = bw * chnz(ichan) / 8. ! Parabolic-shape channel, chnz0: focal height
cxmin = -10.
c -----------------------------------------------------------------
c To check whether it is the first time to route the current channel. If yes,
c we need to find the inflow and outflow at the initial condition (IC)
c from the runon of the channel, and the runoff, subsurface flow, and base

c flow from the related hillslopes.
if(q1(ntchr, ichan) <= -1.e5) then
  nt0 = 0
else
  nt0 = 1
endif
c -----------------------------------------------------------------
c Channel lateral inflow or outflow
c -----------------------------------------------------------------
qlat(0) = qlich(ichan)
do it = nt0, ntchr
  qlat(it) = 0.
enddo
c Base flow contribution from the side hillslopes
c Check for hillslopes to prevent accessing out of bounds index 0
c 2/16/2012 - jrf
areal = 0.0
areat = 0.0
if (nhrght(ielmt).gt.0) areal = areal + hsarea(nhrght(ielmt))
if (nhleft(ielmt).gt.0) areal = areal + hsarea(nhleft(ielmt))
if (nhtop(ielmt).gt.0) areat = areat + hsarea(nhtop(ielmt))
qbasel = cbase * areal
vbasel = qbasel * 86400.
c Total channel lateral inflow calculated by WSHCQI and WSHDRV
volon = rvolat(ielmt) + chnvol(ielmt)
volin = volon - rtrans(ielmt)
c Subsurface flow from the side hillslopes
vsb = tmpsbv(nhleft(ielmt))+tmpsbv(nhrght(ielmt))
c Sum of subsurface flow and base flow
vsbbs = vsb + vbasel
if(volin <= vsbbs) then
c There is lateral outflow when volin < 0, or
c channel lateral inflow comes from base flow and subsurface flow only.
do it = nt0, ntchr
  qlat(it) = volin/86400.
enddo
else
  c Channel lateral inflow comes from base flow, subsurface flow, and surface runoff.
  ih=nhleft(ielmt)
  volhl = 0.0
  volhr = 0.0
  if(ih > 0) volhl = tmpvol(ih)
  ih=nhrght(ielmt)
  if(ih > 0) volhr = tmpvol(ih)
  voltmp = volhl + volhr
  if(voltmp > 0) then
    volhf = (volin - vsbbs) / voltmp
  else
    volhf = 1.0
  endif
  volhl1 = volhl * volhf
  volhr1 = volhr * volhf
  if(nhleft(ielmt) > 0) call chrqin(volhl1, qlat, nt0, 1)
  if(nhrght(ielmt) > 0) call chrqin(volhr1, qlat, nt0, 2)
  do it = nt0, ntchr
    qlat(it) = qlat(it) + vsb/86400. + qbasel
  enddo
endif
volint(ichan) = volint(ichan) + volin

---

c Channel inflow
---

qin(0) = qinich(ichan)
do it = nt0, ntchr
  qin(it) = 0.
enddo
ih=nhtop(ielmt)
volint(ichan) = volint(ichan) + rvotop(ielmt)
if(ih > 0) then
  qbaset = chase * hsarea(ih)
  qsb = tmpsbv(ih) / 86400.
  call chrqin(tmpvol(ih), qin, nt0, 3)
  do it = nt0, ntchr
    qin(it) = qin(it) + qsb + qbaset
  enddo
endif

---

ih = nctop(ielmt)
if(ih > 0) then
  ic = ichid(ih)
  do it = nt0, ntchr
    qin(it) = qin(it) + q1(it, ic)
  enddo
endif
ih = ncrefl(ielmt)
if(ih > 0) then
   ic = ichid(ih)
   do it = nt0, ntchr
      qin(it) = qin(it) + q1(it, ic)
   enddo
endif

c
ih = ncrght(ielmt)
if(ih > 0) then
   ic = ichid(ih)
   do it = nt0, ntchr
      qin(it) = qin(it) + q1(it, ic)
   enddo
endif
qlich(ichan) = qlat(ntchr)
qinich(ichan) = qin(ntchr)

------------------------------------------------------------------
if(q1(ntchr,ichan) < -1.e5) then
  c Calculate the initial condition (IC) of outflow. The IC is assumed to be at steady state.
  q1(0,ichan) = qin(0) + qlat(0)

if(qin(0) > 0.) then
   call Mann(qin(0),ichan,chns0,chnn0,ishp,bw,chnz0,y)
   if(ishp == 1) then
      c Triangular-shape channel
      ain = chnz0 * y * y
   elseif(ishp == 2) then
      c Rectangular-shape channel
      ain = bw * y
   elseif(ishp == 3) then
      c Parabolic-shape channel
      ain = 8./3.*y*sqrt(y*chnz0)
   elseif(ishp >= 4) then
      c Tropezoidal-shape channel
      ain = (bw + chnz0 * y) * y
   endif
   else
      ain = 0.
   endif

if(q1(0,ichan) > 0.) then
   call Mann(q1(0,ichan),ichan,chns0,chnn0,ishp,bw,chnz0,y)
   if(ishp == 1) then
      c Triangular-shape channel
      aout = chnz0 * y * y
   elseif(ishp == 2) then
      c Rectangular-shape channel
      aout = bw * y
   elseif(ishp >= 4) then
      c Tropezoidal-shape channel
      aout = (bw + chnz0 * y) * y
   endif

aout = bw * y
elseif(ishp == 3) then
  c Parabolic-shape channel
  aout = 8./3.*y*sqrt(y*chnz0)
elseif(ishp >= 4) then
  c Tropezoidal-shape channel
  aout = (bw + chnz0 * y) * y
endif
else
  aout = 0.
endif
aavg = 0.5 * (ain + aout)
sfnl(ichan) = aavg * chnl1
lastStor(ichan) = sfnl(ichan)
c -----------------------------------------------------------------
else
  c The initial outflow of today (time 0000) = the final outflow of yesterday (time 2400).
  q1(0,ichan) = q1(ntchr,ichan)
endif
c -----------------------------------------------------------------
do it=0, ntchr
  qs(0, it) = qin(it)
  if(q1(it,ichan) > qmax) qmax = q1(it,ichan)
  if(qin(it) > qmax) then
    qmax = qin(it)
    itpk = it
  endif
  if(qin(it) < qmin) qmin = qin(it)
  qtotl = qin(it) + qlat(it) * 0.5
  if(qtotl < qtmin) qtmin = qtotl
  if(qtotl > qtmax) qtmax = qtotl
endo
df(! KW)
c if(ipeak==3) qref = 0.5 * (qtmin + qtmax)
if(ipeak>=4) qref = 0.5 * (qmin + qmax) ! MC (Ponce and Chaganti, 1994; Ponce et al., 1996; Tewolde and Smithers, 2006)
if(ipeak>=4) qref = 0.5 * (qtmin + qtmax) ! in case there is only lateral inflow but no inflow from top of channel.
do it=0, ntchr
  qlat(it) = qlat(it) / chnl1
endo
call Mann(qref, ichan, chns0, chnn0, ishp, bw, chnz0, y)
ckw = trise * chns0 * sqrt(9.81/y)
if(ckw < 15) then
  print *, 'Warning: may not satisfy KW/DF wave criterion.'
endif
if(ishp == 1) then
c Triangular-shape channel
   ckref = 4.*qref/(3.*chnz0*y*y)
   bw = 2.*y*chnz0
   bt = bw
   aref = chnz0 * y * y
elseif(ishp == 2) then
c Rectangular-shape channel
   bt = bw
   ckref = qref/(bw*y)*(1.+2.*bw/(3.*(bw+2.*y)))
   aref = bw * y
elseif(ishp == 3) then
c Parabolic-shape channel
   bt = 4. * sqrt(chnz0 * y) ! top width
   ap = 2.*sqrt(y*(chnz0+y))+2.*chnz0*log(y/chnz0)
   dqdy = (2.5/y - 4./(3.*ap)*sqrt(1.+bt/y)) * qref
   ckref = dqdy / bt
   aref = 8./3.*y*sqrt(y*chnz0)
elseif(ishp >= 4) then
c Trapezoidal-shape channel
   bt = bw + 2. * chnz0 * y
   sslp = sqrt(1. + chnz0 * chnz0)
   dqdy = (bt*(5.*bw+6.*y*sslp)+4.*chnz0*y*y*sslp)
   dqdy = (bt*(5.*bw+6.*y*sslp)+4.*chnz0*y*y*sslp)
   1 / (3.*y*(bw+chnz0*y)*(bw+2.*y*sslp)) * qref
   ckref = dqdy / bt
   aref = (bw + chnz0 * y) * y
endif
dxchr = dtchr * ckref
if (dxchr.le.0.) dxchr = 1
nseg=chnl1/dxchr
if(nseg > mxcseg) nseg = mxcseg
if(nseg < 1) nseg = 1

c not sure why next line is here

c The following is for the single-spatial-step MC method. It's faster to compute but may lead to negative
outflow for the wave front.
c if(ipeakm == 41) nseg = 1
dxchr = chnl1/nseg
c
do is = 0, nseg
   qs(is,0) = qin(0)+(q1(0,ichan)-qin(0))*float(is)/float(nseg)
enddo

-----------------------------------------------------------------
c Linear kinematic wave method
c
c
   asum = 0.
   if(ipeak == 3) then
      do it=1, ntchr
         if(mofapp==1) then
qlavg = 0.5 * (qlat(it-1) + qlat(it))
else
qlavg = qlat(it)
endif

do is = 1, nseg

qavg = 0.5 * (qs(is,it-1) + qs(is-1,it))
if(abs(qavg) > 0.) then

dtdx = dtchr/dxchr

call Mann(abs(qavg),ichan,chns0,chnn0,
1             ishp, bw, chnz0, y)
if(ishp == 1) then

c Triangular-shape channel
ck = 4.*qavg/(3.*chnz0*y*y)
elseif(ishp == 2) then

c Rectangular-shape channel
ck = qavg/(bw*y)*(1.+2.*bw/(3.*(bw+2.*y)))
elseif(ishp == 3) then

c Parabolic-shape channel
bt = 4. * sqrt(chnz0 * y)
ap = 2.*sqrt(y*(chnz0+y))+2.*chnz0*log(sqrt(1.+y/chnz0)
1 + sqrt(y/chnz0))
dqdy = (2.5/y - 4./(3.*ap)*sqrt(1.+bt/y)) * qavg
ck = dqdy / bt
aref = 8./3.*y*sqrt(y*chnz0)
elseif(ishp >= 4) then

c Tropezoidal-shape channel

sslp = sqrt(1. + chnz0 * chnz0)
dqdy = ((bw+2.*chnz0*y)*(5.*bw+6.*y*sslp)+4.*chnz0*y*y*sslp)
1 / (3.*y*(bw+chnz0*y)*(bw+2.*y*sslp)) * qavg
ck = dqdy / (bw + 2. * chnz0 * y)
c
endif

endif

if (ck < 1e-12) then

qs(is,it) = 0.0
else

cqa = 1./ck

c end change

qs(is,it) = (dtdx*qs(is-1,it) + cqa*qs(is,it-1)
1 + dtchr*qlavg) / (dtdx+cqa)
endif
else

qs(is,it) = qlavg * dxchr
endif
end do

q1(it,ichan)=qs(nseg,it)
if(q1(it,ichan) < eps) q1(it,ichan) = 0.
end do

do is = 0, nseg
if(abs(qs(is,ntchr)) > 0.) then
  call Mann(abs(qs(is, ntchr)), ichan, chns0, chnn0, 
  ishp, bw, chnz0, y)
endif
if(ishp == 1) then
  c Triangular-shape channel
  asum = asum + chnz0 * y * y
elseif(ishp == 2) then
  c Rectangular-shape channel
  asum = asum + bw * y
elseif(ishp ==3) then
  c Parabolic-shape channel
  asum = asum + 8./3.*y*sqrt(y*chnz0)
elseif(ishp >= 4) then
  c Tropezoidal-shape channel
  asum = asum + (bw + chnz0 * y) * y
endif
enddo

aavg = asum / float(nseg + 1)

c -----------------------------------------------------------------

c Muskingum-Cunge method

c elseif(ipeak >= 4) then
  if(qref > 0.) then
    tk = dxchr/ckref
    cx = 0.5*(1.-qref/(bw*ckref*chns0*dxchr))
    if(cx < cxmin) cx = cxmin
  else
    tk = 0.
    cx = 0.
  endif

c0=1./(2.*tk*(1.0-cx)+dtchr)
c1=(dtchr-2.*tk*cx)*c0
c2=(dtchr+2.*tk*cx)*c0
c3=1.-c1-c2
do it=1, ntchr
  qmaxi = max(qin(it-1), q1(it-1,ichan), qin(it))
  if(mofapp==1) then
    qlavg = 0.5 * (qlat(it-1) + qlat(it))
  else
    qlavg = qlat(it)
  endif
  c4=2.*qlavg*dxchr*dtchr*c0
  if(qmaxi > 0. or. qlavg > 0.) then
    do is =1, nseg
      if(ipeak == 5) then ! MVPMC3

qref = (qs(is-1,it)+qs(is-1,it-1)+qs(is,it-1)) / 3.

cw    if(qref > 0.) then
      if(qref < eps) qref = eps
          call Mann(qref,ichan,chns0,chnn0,ishp,bw,chnz0,y)
          if(ishp == 1) then
            c Triangular-shape channel
            ckref = 4.*qref/(3.*chnz0*y*y)
            bw = 2.*y*chnz0
            bt = bw
          elseif(ishp == 2) then
            c Rectangular-shape channel
            bt = bw
            ckref = qref/(bw*y)*(1.+2.*bw/(3.*(bw+2.*y)))
          elseif(ishp == 3) then
            c Parabolic-shape channel
            bt = 4. * sqrt(chnz0 * y) ! top width
            ap = 2. * sqrt(y*(chnz0+y)) + 2. * chnz0
            1 * log(sqrt(1.+y/chnz0) + sqrt(y/chnz0)) ! wetted perimeter
            dqdy = (2.5/y - 4./(3.*ap)*sqrt(1.+bt/y)) * qref
            ckref = dqdy / bt
            aref = 8./3.*y*sqrt(y*chnz0)
          elseif(ishp >= 4) then
            c Tropezoidal-shape channel
            bt = bw + 2. * chnz0 * y
            sslp = sqrt(1. + chnz0 * chnz0)
            dqdy = (bt*(5.*bw+6.*y*sslp)+4.*chnz0*y*y*sslp)
            1 / (3.*y*(bw+chnz0*y)*(bw+2.*y*sslp)) * qref
            ckref = dqdy / bt
          endif
      endif

tk = dxchr/ckref
      cx = 0.5*(1.-qref/(bt*ckref*chns0*dxchr))
      if(cx < cxmin) cx = cxmin
      cw    else
      cw    tk=0.
      cw    cx=0.
      endif
endif

cw

cw    enddo
q1(it,ichan)=qs(nseg,it)
cw    else
q1(it,ichan) = 0.
ENDIF
if(q1(it,ichan) < eps) q1(it,ichan) = 0.
enddo
c
if(qin(ntchr) > 0.) then
call Mann(qin(ntchr),ichan,chns0,chnn0,ishp,bw,chnz0,y)
endif
c
if(ishp == 1) then
c Triangular-shape channel
ain = chnz0 * y * y
endif
c
elseif(ishp == 2) then
c Rectangular-shape channel
ain = bw * y
endif
c
elseif(ishp == 3) then
c Parabolic-shape channel
ain = 8./3.*y*sqrt(y*chnz0)
endif
c
elseif(ishp >= 4) then
c Tropezoidal-shape channel
ain = (bw + chnz0 * y) * y
endif
c
else
ain = 0.
endif

aavg = 0.5 * (ain + aout)
endif
c
sinit(ichan) = sfnl(ichan)
sfnl(ichan) = aavg * chnl1
c

itpk = 0
qchpk = 0.
do it=1, ntchr
if(q1(it,ichan) > qchpk) then
    itpk = it
    qchpk = q1(it,ichan)
endif
end do

---

ih = netop(ielmt)
if(ih > 0) then
    ic = ichid(ih)
    sfnl(ichan) = sfnl(ichan) + sfnl(ic)
endif

ih = ncleft(ielmt)
if(ih > 0) then
    ic = ichid(ih)
    sfnl(ichan) = sfnl(ichan) + sfnl(ic)
endif

ih = ncrght(ielmt)
if(ih > 0) then
    ic = ichid(ih)
    sfnl(ichan) = sfnl(ichan) + sfnl(ic)
endif

c
chvol = volint(ichan) + sinit(ichan) - sfnl(ichan)
if(chvol < 0.) chvol = 0.
if(qchpk <= 0.) chvol = 0.
sfnl(ichan) = sinit(ichan) + volint(ichan) - chvol
qchavg = chvol / 86400.

---

if(volint(ichan) < eps .and. chvol < eps) then
    This was causing water balance to be off, needs to be looked at
    more closely. Without statements water balance is ok. jrf 2-22-2012
endif

---

do i=1,nchnum
    if(ielmt == ichnum(i)) then
        if(ielmt == ichnum(i).and.year==1995 .and. sdate==98) then
            if(ichout < 3) then
                if(ichout == 1)
                    write(66, 104) year, sdate, ielmt, idelmt(ielmt),
                    dtchr*itpk, qchpk
                endif
            endif
        endif
    endif
enddo
write(66, 105) year, sdate, ielmt, idelmt(ielmt), qchavg, chvol
eelseif(qchpk < eps) then
ti=86400.
write(66, 104) year, sdate, ielmt, idelmt(ielmt), ti, qchavg
else
doit=1, ntchr
write(66, 104) year, sdate, ielmt, idelmt(ielmt),
dtchr*it, q1(it, ichan)
enddo
endif
c ccompute the water balance for this day:
c total runon from any hillslopes or channels (rvolon)
c runoff generated by channel alone (chnvol)
c runoff exiting the channel (chvol)
c surface storage for today (sfnl)
c surface storage from previous day (lastStor)
c transmission loss in the channel (rtrans)
bal = rvolon(ielmt)+chnvol(ielmt)-chvol-
(sfnl(ichan)-lastStor(ichan))-rtrans(ielmt)
write(67, 106) year, sdate, ielmt, idelmt(ielmt),
rvolon(ielmt)+chnvol(ielmt), chvol, sfnl(ichan),
qbase(ichan), rtrans(ielmt), bal
endif
enddo
peakot(ielmt) = qchpk
htpk(ielmt) = dtchr * itpk / 3600.
if(chvol <= 0.) then
runvol(ielmt) = 0.0
rundur(ielmt) = 0.0
else
runvol(ielmt) = chvol
rundur(ielmt) = runvol(ielmt)/peakot(ielmt)
endif
runoff(iplane) = runvol(ielmt)/charea(iplane)
tmpvol(ielmt) = runvol(ielmt)
lastStor(ichan) = sfnl(ichan)
write(64, 104) year, sdate, ielmt, idelmt(ielmt),
104 format(1x, 4(i5,2x), 3x, f7.0, 4x, es10.2)
105 format(1x, 4(i5,2x), 2x, es10.2, 2x, f10.2)
106 format(1x, 4(i5,2x), 6(1x, f10.2))
return
end

B.2. CHRQIN.FOR

subroutine chrqin(vol, qin, nt0, iq)
c
++ ++ PURPOSE ++ +
c SR CHRQIN calculates channel inflow (m^3/s) or lateral inflow (m^3/s) for each time step.
Called from: SR WSHCHR
Author(s): L. Wang
Reference in User Guide:
Version:
Date recoded:
Recoded by:

++ + KEYWORDS + + +

++ + PARAMETERS + + +

include 'pmxelm.inc'
include 'pmxhil.inc'
include 'pmxpln.inc'
include 'pmxprt.inc'
include 'pmlxlp.inc'
include 'pmtim.inc'
include 'pmxseg.inc'
include 'pmxcsg.inc'
include 'pmxchr.inc'
include 'pmxtl.inc'
include 'pmxtls.inc'

++ + ARGUMENT DECLARATIONS + + +

++ + ARGUMENT DEFINITIONS + + +

++ + COMMON BLOCKS + + +

include 'cchpek.inc'
include 'cdata1.inc'
include 'cdata3.inc'
include 'chydrol.inc'
include 'cslope.inc'
include 'cstore.inc'
include 'cstruc.inc'
include 'cstruct.inc'
include 'ccvar.inc'
include 'ccpar.inc'
include 'chrt.inc'
include 'cupdate.inc'

++ + LOCAL VARIABLES + + +

real a1, b1, d1, u, vol, tc, td, ti, qp, eps, qin(0:mxtchr),
1 tl, expap1, expap2, pksu(3), qin0(0:mxtchr)
integer it, ierr, iq, icnt, ieltmp(3), nt0

135
+ + + LOCAL DEFINITIONS + + +

a1 - constant in the equation: 1 - exp(-u) = a1 * u
in this subroutine, a = vol/(qp * td)
b1 - coefficient in double exponential
d - coefficient in double exponential
ierr - Flag. 0: equation solved
1: no solution for given a
it - current time step number
qin - channel inflow rate, [m3/s]
qp - peak runoff of overland flow, [m3/s]
tc - time of concentration of overland flow, [s]
td - runoff duration of a hillslope, [s]
u - variable in the equation: 1 - exp(-u) = a1 * u
vol - runoff volume of a hillslope, [m3]

+ + + SUBROUTINES CALLED + + +
eqroot

+ + + DATA INITIALIZATIONS + + +

+ + + END SPECIFICATIONS + + +

c = 1.e-6
vsum = 0.0
if(vol > c) then

if(iq == 1) then ! lateral inflow from left hillslope
td = watdur(nhleft(ielmt))
qp = tmppkr(nhleft(ielmt))
elseif(iq == 2) then ! lateral inflow from right hillslope
td = watdur(nhrght(ielmt))
qp = tmppkr(nhrght(ielmt))
else ! inflow from top hillslope
td = watdur(nhtop(ielmt))
qp = tmppkr(nhtop(ielmt))
endif
tc = htc(ielmt)*3600.
tc = tc / 2.67

if(vol >= qp*td-c) then
do it = nt0, ntchr
ti = dtchr * it
if(ti <= td) then
    qin(it) = qin(it) + qp
endif
enddo
else
  a1 = vol / (qp * td)
call eqroot(a1, ierr, u)
b1 = u / tc
d1 = u / (td - tc)
c--------------------------------------------------
c using pointwise value
  if(mofapp==1) then
do it = nt0, ntchr
ti = dtchr * it
  if(ti <= tc) then
    qin0(it) = qp * exp(b1*(ti-tc))
vsum = vsum + qin0(it)
  elseif(ti <= td) then
    qin0(it) = qp * exp(d1*(tc-ti))
vsum = vsum + qin0(it)
endif
dendo
c checking mass balance in case time step too large
vsum = (vsum - qin0(nt0)) * dtchr
vf = vol / vsum
do it = nt0, int(td/dtchr)
  qin(it) = qin(it) + qin0(it) * vf
dendo
c--------------------------------------------------
c using cell average
else
do it = nt0, ntchr
ti = dtchr * it
  if(u <= eps) then
    if(ti <= td) qin(it) = qin(it) + qp
  elseif(ti < tc) then
    tl = ti - dtchr
    if(b1 <= eps) then
      qin(it) = qin(it) + qp*(1.+(ti-tc-0.5*dtchr)*b1)
    elseif(b1 <= eps) then
      expap1 = tc-tl-0.5*b1*(tl-tc)**2
    else
      expap1 = (1.-exp(b1*(tl-tc)))/b1
      qin(it) = qin(it) + qp*exp(expap1*(ti-tc**2))
  endif
dendo
elseif(tl < tc) then
  if(b1 <= eps) then
    expap1 = tc-tl-0.5*b1*(tl-tc)**2
  else
    expap1 = (1.-exp(b1*(tl-tc)))/b1
  endif
  if(d1 <= eps) then
    expap2 = tc-ti+0.5*d1*(ti-tc)**2
  endif
else
    expap2 = (exp(-d1*(ti-tc))-1.)/d1
endif
qin(it) = qin(it) + qp*(expap1 - expap2)/dtchr
elseif(tl < td) then
    if(d1 <= eps) then
        qin(it) = qin(it) + qp*(1.-(ti-tc-0.5*dtchr)*d1)
    else
        qin(it) = qin(it) + qp*((exp(-d1*(tl-tc)) - exp(-d1*(ti-tc)))/(d1*dtchr))
    endif
endif
enddo

B.3. MANN.FOR

subroutine mann(q0, ichan, slp, chnn, ishp, bw, s, y)

    implicit none
    c
    c        + + + PURPOSE + + +
    c    SR Mann solves depth y from Manning's equation.
    c
    c    Called from: SR WSHCHR
    c    Author(s): L. Wang
    c    Reference in User Guide:
    c
    c    Version:
    c    Date recoded:
    c    Recoded by:
    c
    c        + + + KEYWORDS + + +
    c
    c        + + + PARAMETERS + + +
    c
    c        + + + ARGUMENT DECLARATIONS + + +
    real q0, slp, chnn, y, bw, s
    integer ichan, ishp
    c
    c        + + + ARGUMENT DEFINITIONS + + +
    c
    c    bw    - channel width
c chnn - Manning's roughness coefficient n
c q0 - known discharge
c s - inverse slope of the channel banks
c slp - channel slope
c y - depth of water flow
c ichan - channel number
c ishp - shape of the channel

c
+ + + COMMON BLOCKS + + +

c + + + LOCAL VARIABLES + + +

c real area, rh, ay, ry, qi, dy, q0x, yx, eps, ap
integer i, icanx

c
c + + + LOCAL DEFINITIONS + + +

c area - cross-sectional area of channel water flow
c ay - derivative, AY = dA / dY
c qi - calculated discharge
c rh - hydraulic radius
c ry - derivative, RY = dR / dY
c icanx - channel number

c + + + SAVES + + +
save icanx, q0x, yx

c + + + SUBROUTINES CALLED + + +

c + + + DATA INITIALIZATIONS + + +

c + + + END SPECIFICATIONS + + +

c
eps = 1.e-6
if(icanx /= icanx .or. abs(q0 - q0x) > eps)then

if(ishp == 1) then

c Triangular-shape channel

c Calculate y explicitly.
c
  y = 2.**0.25*(1.+s*s)**0.125*(chnn*q0)**0.375/
     1     (slp**0.1875*s**0.625)

c
elseif(ishp == 2) then

c
c Rectangular-shape channel
c Calculate y iteratively using Newton's method.
c
c First guess of y assuming bw >> y.
c y = (chnn*q0/(sqrt(slp)*bw))**0.6
 y = 1.
c
i = 0
400 i = i + 1
 area = bw * y
 rh = area / (bw + 2. * y)
 ay = bw ! ay = dA / dy
 ry = (bw / (bw + 2. * y))**2 ! ry = dR / dy
 qi = sqrt(slp) / chnn * area * rh ** (2./3.)
 dy = (q0/qi-1.) / (ay/area + 2./3.*ry/rh)
 y = y + dy
 if(y > 1) dy = dy / y
 if(abs(dy) > eps .and. i < 20) goto 400

 controlled by non-dimension

 c--------------------------------------------------
                  c--------------------------------------------------
      elseif(ishp == 3) then

 c Parabolic-shape channel
c Calculate y iteratively using Newton's method.
c s = parabolic focal height
c
 y = 1.
c
i = 0
300 i = i + 1
 bw = 4. * sqrt(y * s) ! top width
 area = 2. * bw * y / 3.
 ap = 2.*sqrt(y*(s+y))+2.*s*log(sqrt(1.+y/s)+sqrt(y/s)) ! wetted perimeter
 rh = area / ap ! hydraulic radius
 ay = bw ! ay = dA / dy
 ry = bw/ap - 2.*area/(ap*ap)*sqrt(1.+s/y) ! ry = dR / dy
 qi = sqrt(slp) / chnn * area * rh ** (2./3.)
 dy = (q0/qi-1.) / (ay/area + 2./3.*ry/rh)
 y = y + dy
 if(y > 1) dy = dy / y
 if(abs(dy) > eps .and. i < 20) goto 300

 c--------------------------------------------------
                  c--------------------------------------------------
      elseif(ishp >= 4) then

 c Tropezoidal-shape channel
c Calculate y iteratively using Newton's method.
c
 y = 1.
i = 0
200    i = i + 1
    area = (bw + s * y) * y
    rh = area / (bw + 2. * y * sqrt(1. + s * s))
    ay = bw + 2. * s * y
    ry = (bw*bw + 2.*bw*s*y + 2.*s*y*y*sqrt(1.+s*s))
        / (bw + 2. * y * sqrt(1. + s * s))**2
    qi = sqrt(slp) / chnn * area * rh ** (2./3.)
    dy = (q0/qi-1.) / (ay/area + 2./3.*ry/rh)
    y = y + dy
    if(y > 1) dy = dy / y
    if(abs(dy) > eps .and. i < 20) goto 200

B.4. PMXCHR.INC

c     begin include file pmxchr.inc

c     + + + PARAMETER DECLARATIONS + + +

    integer mxtchr
    real    dtlowl, dtupl1, dtupl2

    parameter (mxtchr=1440, dtlowl=60., dtupl1=3600., dtupl2=1800.)

c     + + + PARAMETER DEFINITIONS + + +

c     mxtchr - maximum number of time steps for channel routing
c     dtlowl - lower limit of time step [s]
c     dtupl1 - upper limit of time step for continuous modeling [s]
c     dtupl2 - upper limit of time step for event modeling [s]

    endif

return
end
B.5. CCHRT.INC

c begin include file cchrt.inc

c +++++ COMMON BLOCK DESCRIPTION +++++

c +++++ INSERT DESCRIPTION HERE +++++

common /chrt/ichout,ntchr,nxchr,nchnum,mofapp,dtchr,dxchr,
1       ichid(mxelem),ichnum(mxplan),q1(0:mxtchr,mxplan),
1       sinit(mxplan),volint(mxplan),sfnl(mxplan),cbase,
1       htpk(0:mxelem),qlich(mxplan),qinich(mxplan),trise,
1       hsarea(mxhill),qBase(mxplan),lastStor(mxplan)

c +++++ VARIABLE DECLARATIONS +++++

integer  ichout,ntchr,nxchr,nchnum,ichid,ichnum,mofapp
real     dtchr,dxchr,q1,sinit,volint,sfnl,cbase,htpk,qlich,
1       qinich,trise,hsarea,qBase,lastStor

c +++++ VARIABLE DEFINITIONS +++++

ichid  : channel number corresponding to its element ID number
c ichnum : channel IDs for channel routing output
c ichout : flag for channel flow output
   0, no output
c 1, output peak flow time and rate for specified channels
c 2, output daily average flow rate and total runoff for specified channels
c 3, output flow rate at each time step for specified channels
mofapp : method of approximation for qin and qlat, 1 or 2
nchnum : number of channels for channel routing output
ntchr : number of time steps
nxchr : number of space intervals
dtchr : time step [s]
dxchr : space increment [m]
sfnl : final storage of the channel system above an outlet [m^3]
sinit : initial storage of the channel system above an outlet [m^3]
trise : time of rise [s]
volint : daily inflow volume to the channel system above an outlet [m^3]
cbase : unit area base flow [m3/s/m2]
qBase : Baseflow channel [m^3]
lastStor : previous days surface storage [m^3]

c end include file cchrt.inc