NUMERICAL MODELLING OF THERMOACOUSTIC HEAT PUMPS AND PRIME
MOVERS WITH NO STACKS AND INTERMITTENT STACKS

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Abstract

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Thermoacoustic systems with no stacks or intermittent stacks represent a promising alternative to conventional stack-based systems. There is theoretical and experimental evidence that such systems can perform more efficiently and produce more acoustic power than thermoacoustic systems with an array of flat plates or regular pores.

As opposed to conventional thermoacoustic systems, no-stack setups do not include porous material as a medium for storing and transferring heat. In such systems, heat is transported by gas parcels directly between heat exchangers. A simplified Lagrangian model has been developed in this work to approximately analyze no-stack standing-wave thermoacoustic configurations. More accurate simulations of favorable configurations are investigated with the help of computational fluid dynamics (CFD). Using CFD code FLUENT, more efficient setups were found with the second law efficiency up to 55%. The dependence of system performance is reported for a range of system parameters. A good agreement between the simplified model and CFD results was achieved in most cases. Heat exchangers with finite-thickness plates were additionally investigated in high-performance setups. For porosities below 25%, the thickness effect appeared to be insignificant.
In the other part of this study, a reduced-order model was applied for examining a transverse-pin array stack thermoacoustic system. This configuration can be more efficient than conventional stack systems by reducing the heat loss through conduction in the direction of acoustic oscillations and by increasing the contact surface between the working fluid and the solid material. Also, finite heat capacity of the pins was accounted for in this model. Effects of different thermal and geometrical parameters on the performance of the setup were investigated. The numerical model can be used as a tool to optimize such systems.
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Dedication

This dissertation is dedicated to my mother, my father, and my siblings for their emotional and financial support.
CHAPTER 1

INTRODUCTION
Introduction

Thermoacoustics is the field of study about sound and heat interactions. Thermoacoustic engine technology utilizes high-amplitude sound waves in a pressurized gas to pump heat from one place to another (in heat pumps) or uses a temperature difference to induce oscillating pressure or sound waves (in prime movers), which can be utilized for other applications. These systems have no ozone-depleting or toxic coolants and few moving parts.

Rayleigh criterion (Rayleigh, 1945) can explain the sound generation in thermoacoustic prime movers. According to Rayleigh criterion, the addition of heat to the oscillating gas in which the fluctuation pressure is in phase with the heat introduced to the system will result in the generation of acoustic energy. Also, this criterion implies that the generated thermoacoustic power $\dot{E}_{TA}$ is proportional to the following integral taken over the chamber volume, $V$ and over one acoustic cycle:

$$\dot{E}_{TA} \approx f \int_0^T \int_V p'(x,t)\dot{q}'(x,t)dVdt$$  \hspace{1cm} (1.1)

where $f$ is the frequency of oscillation, $p'$ and $\dot{q}'$ are respectively the fluctuating pressure and the heat transfer rate per unit volume.

The core of many thermoacoustic devices is the stack, which can comprise a collection of closely spaced plates or an array of channels across which a temperature gradient is imposed. The stack is fixed in place at a set location inside a resonator. The stack is usually sandwiched between permeable hot and cold heat exchangers that create a temperature gradient. In a simple standing-wave resonator shown in Fig. 1.1, the hot heat exchanger is near the closed end and the
cold one is by the open end. Temperatures of the stack ends are maintained by the heat exchangers. The basic principles of thermoacoustic engines can be illustrated as follows: if a gas as the medium for an acoustic standing wave and a solid plate parallel to the direction of acoustic motions are present in a tube (resonator), then the standing wave is modified by the presence of the plate which leads to two major effects: (1) there is a time-average heat flux along the plate over one period and (2) acoustic power can be generated or consumed around the plate. In practical systems, thermoacoustic power conversion takes place in porous materials, such as a stack of parallel plates (Swift, 1988). Fluid parcels going back and forth carry heat from one heat exchanger to the other. The resonator geometry and fluid properties determine the frequency of oscillations (Swift, 1988).

![Figure 1.1 Stack-based thermoacoustic heat engine](image)

One role of the stack is to transfer heat from one parcel of the oscillating gas to the next parcel. In this process, heat is transferred from a parcel to the stack and then from the stack to the adjacent parcel. Also, the presence of stack introduces phase shift between oscillating velocity and oscillating pressure which is results in acoustic power generation.

Many thermoacoustic systems use standing waves. A standing wave – also known as a stationary wave – is a waveform that remains in a constant position. Standing waves are natural phenomena exhibited by waves of different nature, such as sound, light, or even water waves. These waves in a gas are basically pressure waves. Standing waves seem to vibrate in constant
position around stationary nodes. Where the maximum displacement occurs is called the antinode. The maximum compression of the gas also occurs at the pressure antinodes. In the fundamental acoustic mode, the pressure and velocity can be modeled using sine and cosine functions as follows:

\[ p'(x, t) = P_a \cos kx \cos \omega t \]  \hspace{1cm} (1.2)

\[ p(x, t) = p_m + P_a \cos kx \cos \omega t \]  \hspace{1cm} (1.3)

\[ u'(x, t) = -\frac{P_a}{\rho_m a} \sin kx \sin \omega t \]  \hspace{1cm} (1.4)

where \( p \) is the absolute pressure inside the resonator, \( p' \) and \( u' \) are the fluctuating pressure and velocity respectively, \( p_m \) is the mean pressure, \( P_a \) is a constant number whose unit is the same as pressure and shows the maximum value of oscillating pressure (it is also known as the pressure amplitude at the closed end), \( k = \pi/2L \) is the wave number, \( \omega = \pi a/2L \) is the angular frequency, \( x \) is the displacement variable which is the distance from the closed end towards the open end in Fig. 1.1, \( t \) is the time, \( \rho_m \) is the fluid mean density, and \( a \) is the speed of sound. The fundamental acoustic modes inside a resonator, shown in Fig. 1.2, describe primary fluctuations of acoustic pressure and velocity.
Figure 1.2 The sinusoidal change in the amplitude of oscillating pressure and velocity inside the resonator

Figure 1.2 shows how the amplitude of oscillating pressure and velocity inside a resonator change. At the closed end of the resonator, the left end, the pressure amplitude is maximum which means pressure is oscillating with this maximum amplitude at the wall. At the open end the pressure amplitude is zero which means there is no pressure oscillation at the open end. On the other hand it is the opposite for the velocity amplitude. At the closed end, the wall, velocity amplitude is zero because of rigid-wall boundary condition but at the open end the amplitude of oscillatory velocity is maximum. In addition, there is a phase shift between velocity and pressure in Fig. 1.2 which is $\varphi = 90^\circ$ in this particular case.

As mentioned before, a standing wave thermoacoustic prime mover is capable of generating acoustic power. Swift (1988) showed that the acoustic power can be expressed by the following formula. This expression was developed for an inviscid ideal gas oscillating at the two sides of a solid plate along which a temperature gradient of $\nabla T$ is imposed:

$$\dot{W}_2 = \frac{1}{2} \delta_k B L \omega \frac{\gamma - 1}{\gamma} \frac{p_1}{p_m} \left( \frac{\nabla T}{\nabla T_{cr,ld}} - 1 \right)$$  \hspace{1cm} (1.5)
where $B$ is the width of the plate, $L$ is the length of the plate, $p_1$ is the complex pressure amplitude of oscillation, $\nabla T_{cr,1d}$ is the critical temperature gradient for an idealized system. $\nabla T_{cr,1d}$ is the threshold of acoustic power generation in an idealized system. (Swift, 1988):

$$\nabla T_{cr,1d} = \frac{\omega p_1}{\rho_m C_p u_1} \quad (1.6)$$

where $u_1$ is the complex velocity amplitude and $C_p$ is the constant pressure specific heat of the gas. $\delta_k$ is the thermal penetration depth which is a measure of the distance that heat can penetrate in a gas over one acoustic cycle:

$$\delta_k = \sqrt{\frac{2k}{\rho_m C_p \omega}} \quad (1.7)$$

$k$ is the conductivity of the gas. For efficient engine performance the distance between two plates of the stack should be a few thermal penetration lengths. In real systems $\nabla T$ should be greater than $\nabla T_{cr,1d}$ to be able to generate power. To calculate actual $\nabla T_{cr}$ one would need to balance acoustic power production in the stack with the sum of all acoustic losses in the entire system (Swift, 1988; Jung and Matveev, 2010).

The role of the stack in conventional thermoacoustic systems is to act as a medium to interact with the working gas parcels. For example in thermoacoustic engines, the air at the hot end of the stack is not as warm as the surrounding walls, so heat flows into the air parcel. As the air warms, it expands and the parcel moves towards the cold end. As it travels, it cools, but the surrounding stack is colder still, therefore heat travels from the air to the stack. Since the air is now cooler, it contracts, and moves back towards the hot end to repeat the cycle. The next parcel picks up the heat from the stack and moves towards the cold end; gives the heat to the stack and
moves back. Basically, the stack and gas parcels act as a bucket brigade together to pump the heat from the hot end to the cold end. Using parallel plates or other porous materials as stacks or regenerators has been a conventional way to construct thermoacoustic engines or refrigerators. On the other hand, there are potentially more efficient systems with different patterns for the stack or even without a stack. A new thermoacoustic setup proposed by Wakeland & Keolian (2002) involves two heat exchangers but no stack. In this system, the distance between the heat exchangers is much smaller than that of the conventional thermoacoustic system because oscillating parcels of the gas are supposed to carry the heat directly between heat exchangers (Fig. 1.3).

![Diagram](image.png)

**Figure 1.3** a) No-stack thermoacoustic system. b) Stack-based thermoacoustic system

One can pose a question: Is this system more efficient? One possible advantage is the elimination of the stack that may decrease friction loss. Wakeland & Keolian (2004) studied this setup using a simplified Lagrangian numerical model and showed that it could work more efficiently when operating at high pressure amplitudes. The reason they employed this mathematical approach is that the commonly used low-amplitude thermoacoustic theory (Swift, 2002) is not able to describe the no-stack thermoacoustic devices. Their model focused only on the parcels which travel between both heat exchangers and used simplified equations for the heat
transfer between the gas and the heat exchangers. Also, the acoustic pressure, velocity and displacement were assumed to oscillate sinusoidally. As was shown in the earlier paper by the same authors (Wakeland & Keolian, 2002), an efficient no-stack refrigerator needs greater drive ratios (higher pressure amplitudes) at which second-order effects become more significant and cannot be ignored in accurate calculations. One of our motivations is to develop potentially more efficient thermoacoustic configurations with the help of numerical methods such as simplified models and more detailed computational fluid dynamics codes (CFD).

A number of papers were published on CFD simulations of thermoacoustic phenomena. Cao et al. (1996) solved compressible viscous fluid equations for a thermoacoustic couple using CFD and presented energy flux streamlines on a stack plate in a simple thermoacoustic system. Just like other methods, CFD modeling offers some advantages and disadvantages. It can be very expensive considering the fact that for some cases dense mesh and small time steps are required. Toffolo et al. (2010) presented some guidelines to reduce the computational effort needed to perform a CFD analysis of thermoacoustic oscillations such as grid refinement and the smallness of time steps in modeling oscillations of high frequency. On the other hand, nonlinear phenomena that the existing linear theory cannot capture like streaming flows and vortices formation can be visualized and studied by CFD. Lycklama et al. (2005) studied a travelling wave thermoacoustic engine using CFD. They used a simplified geometry for the regenerator and concluded that CFD codes can be used to predict and optimize a thermoacoustic system which is also the aim of this paper. Hantschk et al. (1999) employed commercial CFD code FLUENT to simulate self-excited thermoacoustic instabilities in a Rijke tube and obtained results that were in good agreement with experiments. As mentioned before, one advantage of CFD is that the nonlinear effects can also be accounted for and this was one of their incentives to use
CFD. Another advantage of commercial CFD codes is that more complicated geometries can be generated more easily. For example, Zink, Vipperman, and Schaefer (2010) looked at the influence of resonator curvature on the thermoacoustic effect. Using FLUENT enabled them to show that the presence of the curvature in the resonator influences both the amplitude and the frequency of sound waves.

In chapter 2, we will derive the necessary equations for the simplified numerical model of no-stack thermoacoustic systems. Chapter 3 presents the results of studying no-stack configurations from the simplified numerical model and CFD.

We should emphasize that heat can only diffuse and be convected a finite distance through the gas in one period of acoustic cycle. These distances, the thermal penetration length and the acoustic displacements, are usually much smaller than the distance between the heat exchangers, which makes it necessary to apply plates or other solid matrix in between. However, instead of uniform parallel plates, as in usual configurations, stack can also take form of other solid porous structures introduced to maximize the participation of gas in pumping heat effectively. In other words, in order for heat to be pumped from one end to the other, some organized or random narrow channels must be present through which the gas parcels can exchange heat with the solid material.

A number of papers have proposed non-uniform stacks also known as intermittent stacks. Bosel et al. (1999) replaced parallel plate stack with parallel plate segments. The length of these segments was smaller than the acoustic displacement amplitude and they were oriented randomly to each other. Bosel’s goal was to increase the heat transfer rate between the heat exchangers as well as the power density. Also since the parallel plate segments were much shorter than regular
stack place heat conduction in the direction of the flow could be reduced in such configuration. They developed a simplified numerical model to analyze this system. Their theoretical results were backed up by experimental results from an apparatus and both results were in qualitative agreement. In addition both results showed that intermittent stacks lead to bigger power density and better efficiency or coefficient of performance. Petculescu (2002) looked at an array of pins as the stack with two major orientations, longitudinal-pin array and transverse-pin array stacks. For the longitudinal orientation, pins are aligned such that their axes are parallel to the direction of the flow. For the transverse orientation pins’ axes are perpendicular to the direction of the flow. She studied how oscillatory flow interacts thermally and viscously with uniform and non-uniform stacks. Her work showed that both configurations demonstrated better power density compared to circular pores as the stack. Also according to his results, the transverse-pin setup is more advantageous because it reduces conduction losses by minimizing the path between the heat exchangers to only that between the heat exchangers and the pin-array. Swift (2002) showed that conduction in the direction of acoustic flow both in the working fluid and in the stack is not conducive to the amount of heat pumped from one end to the other. This accounts for the fact that Petculescu (2002) found the transverse-pin array stack more efficient than the longitudinal orientation. In addition, the pins at both ends of the stack can be used as heat exchangers by running cold and hot fluid through hollow pins. Matveev (2010) developed a simplified numerical model for transverse-pin array stacks. He compared the efficiency of this system with that of an equivalent parallel plate stack system and numerically found it more efficient. In his study, he assumed that the heat capacity of the solid material is sufficiently larger than that of the working gas so that the temperature fluctuation inside the pins can be neglected. Therefore, in his model, every pin’s temperature is constant. In reality, for solid materials with low heat capacity,
the interaction between thermoacoustic flow and pins causes the pins’ temperature to fluctuate with time. This temperature fluctuation can change the amount of acoustic power and heat transfer rates, thereby changing the efficiency of the system. In chapter 4, we will develop a simplified model based on Matveev’s model (2010). In addition, we account for temperature fluctuation inside the pins. The motivation to develop a simplified model for this configuration is the fact that similar to no-stack systems, there is no available theoretical solution for a transverse-pin stack thermoacoustic system. As stated by Petculescu (2002), another potential advantage of non-uniform stack is that it can reduce the amount of heat loss through conduction in pins and provides more contact surface since they are oriented perpendicular to the acoustic motion.

In recent years, the use of numerical modeling to study thermoacoustic phenomena has grown. Rulik et al. (2011) used the commercial CFD code CFX 12 to model a thermoacoustic engine. Their model was qualitatively similar to the model used in this thesis for validation in chapter 3. They looked at the flow around one of the stack plates along which a temperature gradient is imposed. In their model, the plate had a finite thickness and the vortices around the plate ends were captured. Besides the vortices, complex temperature patterns in the vicinity of stack ends were determined. Berson et al. (2011) employed numerical analysis to investigate temperature field around the stack ends using a relaxation time approximation. They studied the effect of axial conduction on temperature fluctuations. Their work consisted of solving the compressible energy equation while accounting for conduction heat transfer in the axial direction. CFD has proved to be a reliable method for modeling thermoacoustic processes. For example, Song et al. (2011) predicted instabilities in a Rijke tube and showed that their results were in good agreement with experimental data. Some studies were dedicated to finding most
reliable discretization schemes for thermoacoustic flow phenomena. Jiping et al. (2007) used different methods to discretize convection-diffusion terms in momentum and energy equation and discussed the accuracy of those schemes.

The main objectives of this work are (1) to advance knowledge about thermoacoustic engines with no-stack configurations and intermittent stacks, (2) to develop simplified models for fast parametric analysis and optimization of such systems, (3) to conduct more detailed but more costly computational fluid dynamics simulations for determining applicability of the simplified analysis and obtaining more accurate results in parametric studies.
CHAPTER 2

NO-STACK MODELS
No-Stack Models

Our motivation is to find potentially more efficient thermoacoustic heat pumps and prime movers. In chapter 1 it was mentioned that Wakeland and Keolian (2002) proposed the idea of a no-stack system (Fig. 1.3). Their theoretical model was based on the fact that if the distance between the two heat exchangers is small enough, there is no need for a stack. This is because the stack’s job is to extract heat from one parcel of gas and transfer it to the next parcel. Therefore, if the distance is small enough, the parcel itself can transfer heat from the hot source to the cold one in the case of engines and vice versa in the case of refrigerators (Fig 2.1).

![Figure 2.1 No-stack thermoacoustic system: if the distance between the heat exchangers is small enough, gas parcels can transfer heat.](image)

First, they proposed that a gas parcel goes through a Brayton cycle between the hot and cold heat exchangers (Fig. 3 from Wakeland and Keolian, 2002). The second law efficiency of such heat pump without considering losses was around 60%. Considering losses and introducing some expressions for the heat transfer rate and power in one cycle, finally they derived an expression for the second-law efficiency which was more realistic. For some specific configuration, they calculated a second-law efficiency of 37% at the drive ratio of 26%. Drive ratio and second law efficiency can be defined as follows:

$$DR = \frac{P_A}{P_m}$$

(2.1)
\[ \eta_{II(Engine)} = \frac{\eta_{th}}{\eta_C} \]  
(2.2)

\[ \eta_{II(Heat Pump)} = \frac{COP}{COP_C} \]  
(2.3)

where \(\eta_{th}\) is the thermal efficiency of an engine, \(\eta_C\) is the Carnot efficiency of the same engine, \(COP\) is the coefficient of performance of a heat pump, and \(COP_C\) is the Carnot coefficient of performance of the same heat pump. Full derivations of these parameters for no-stack thermoacoustic systems will be shown later.

In another attempt, Wakeland and Keolian (2004) introduced a simplified model which was based on tracking gas parcels. Several simplified numerical models (e.g., Wakeland and Keolian, 2004; Matveev, 2010) for thermoacoustic phenomena share the method of chasing gas parcels. In some of these models, it is assumed that the acoustic pressure is only a function of time not a function of \(x\), where \(x\) is the position of the parcel in the resonator with respect to the closed end as \(x = 0\) (Fig 2.1). The reasoning behind this approach is that we usually deal with a small portion of the resonator, which is much smaller than the acoustic wavelength (Swift, 2002). In this model, they chose not to assume that the heat exchangers were perfect, but they defined a time delay parameter. That is, it takes the gas parcel some time to reach the temperature of the heat exchanger and since the gas parcel is oscillating with some frequency, depending on the frequency, the gas parcel might or might not reach the temperature of the heat exchanger.

Using this new model, they proved that the second-law efficiency for the same system is actually lower but reducing heat exchangers’ effectiveness can actually increase the amount of heat pumped by a no-stack heat pump. In their paper, they showed how different “time delay
parameters” can affect the efficiency and heat transfer rate. The problem is that there is no reliable “time delay parameter” that we can actually use although they proposed a way to calculate the time delay parameter as a function of frequency in Fig 8 of their paper.

Their work suggested that there could be different geometries for pumping the heat between exchangers apart from the conventional parallel plates. We already know that the theory we have access to is only valid for low-amplitude oscillations with parallel plate stacks, circular pores, rectangular channels and pins oriented along the direction of acoustic oscillations. There are no accurate analytical solutions for no-stack systems and more innovative geometries, especially with high amplitudes. This was our motivation to develop a more comprehensive simplified model for no-stack systems based on their model to predict potentially more efficient systems and then use CFD and full numerical solution of Navier-Stokes equations to solve the flow field for the optimum systems that our simplified numerical model suggests.

**Simplified Numerical Model**

In our simplified numerical model we assume the pressure is changing sinusoidally and since we look at a small portion of the resonator it is only a function of time not spatial variables. Therefore, pressure can be expressed using the following equation:

\[
p(t) = p_m + p_A \sin \omega t
\]

(2.4)

Our model for gas parcels’ velocity and position must be such that no-slip boundary condition at the wall is satisfied. That is, the velocity at the wall must be zero and the parcel adjacent to the wall does not move and farther parcels travel longer distances. We use the adiabatic approximation which was also suggested by Wakeland and Keolian (2004):
\[ p(t)x_i(t)' = p_m x_{0,i}' \]  

(2.5)

where \( x \) is the position of the parcel in the resonator with respect to the closed end as \( x = 0 \) (Fig 2.1) and \( x_{0,i} \) is the initial position of \( i \)th parcel at \( t = 0 \), which is simply the parcel position when the fluid is at rest and its pressure, \( p_m \), is uniform throughout the channel. \( x_i(t) \) is the position of \( i \)th parcel at time \( t \). Although there is heat transfer between heat exchangers and parcels, this assumption works well in predicting the position of every parcel because the length of heat exchangers is much smaller than the studied area. From Eq. 2.5 \( x_i(t) \) can be found for every parcel:

\[ x_i(t) = x_{0,i} \left[ \frac{p_m}{p(t)} \right]^{\frac{1}{\gamma}} = x_{0,i} \left[ \frac{p_m}{p_m + p_A \sin \omega t} \right]^{\frac{1}{\gamma}} = x_{0,i} \left[ \frac{1}{1 + D R \sin \omega t} \right]^{\frac{1}{\gamma}} \]  

(2.6)

By definition, velocity of every parcel \( u_i(t) \) can be calculated by taking the derivative of \( x_i(t) \) with respect to time:

\[ u_i(t) = \frac{dx_i(t)}{dt} = -\frac{1}{\gamma} x_{0,i} \left[ \frac{1}{1 + D R \sin \omega t} \right]^{\frac{1}{\gamma}} \left[ \frac{D R \omega \cos \omega t}{1 + D R \sin \omega t} \right] \]  

(2.7)

So far, we know the pressure, the position, and the velocity of every gas parcel at every time step for one acoustic cycle. The goal is to find the change in temperature of the parcel during a cycle. The temperature is more susceptible to the second-order thermoacoustic effects. Most importantly, temperature variation of every parcel contributes to heat transfer between the parcels and the heat exchangers. Time-average heat transfer rates over a cycle are required to find efficiency or coefficient of performance.
In order to find temperature, we write the first law of thermodynamics for a closed system. The closed system here is a gas parcel:

\[
\frac{dQ}{dt} - \frac{dW}{dt} = \frac{dE_{sys}}{dt}
\]  \hspace{1cm} (2.8)

where \( Q \) is the net heat transfer into the system, \( W \) is the net work going out of the system, and \( E_{sys} \) is the internal energy of the system. We are looking at a time interval between \( t \) and \( t + dt \). There are two types of heat transfer for every parcel: (1) heat transfer between the parcel and the heat exchangers, (2) heat transfer between every parcel and its adjacent parcels. In our simplified model, the heat transfer from other parcels is neglected and the only heat transfer considered is between the heat exchangers and the parcels. Therefore, the first law of thermodynamics can be rewritten as follows:

\[
\frac{dQ_{HX}}{dt} - \frac{dW}{dt} = \frac{dE_{sys}}{dt}
\]  \hspace{1cm} (2.9)

\( dQ_{HX} \) represents the heat transfer between the parcel and heat exchangers. Using \( dW = pdV \) and \( dE_{sys} = m_p C_v dT \) where \( dV \) is the change in the volume of the parcel, \( m_p \) is the mass of the parcel, \( C_v \) is the constant volume specific heat of the gas, and \( dT \) is the change is the temperature of the parcel, the first law will be:

\[
\frac{dQ_{HX}}{dt} - pdV = m_p C_v dT
\]  \hspace{1cm} (2.10)

Volume can be expressed as a function of temperature and pressure using the ideal gas law for a single parcel:
\[ pV = m_pRT \Rightarrow d(pV) = d(m_pRT) \Rightarrow pdV + Vdp = m_pRdT \Rightarrow pdV \]
\[ = m_pRdT - Vdp \]  
(2.11)

where \( R \) is the ideal gas constant. Also:

\[ V = \frac{m_pRT}{p} \]  
(2.12)

Therefore:

\[ pdV = m_pRdT - \frac{m_pRT}{p} dp \]  
(2.13)

Substituting Eq. 2.13 into Eq. 2.10 and multiplying both sides by \( dt \):

\[ dQ_{HX} - m_pRdT + \frac{m_pRT}{p} dp = m_pC_vdT \]  
(2.14)

From Eq. 2.14 the change in temperature can be calculated:

\[ \left( \frac{C_v}{R} + 1 \right) dT = \frac{dQ_{HX}}{m_pR} + \frac{T}{p} dp \]  
(2.15)

\[ \frac{C_v}{R} + 1 = \frac{C_p}{R} = \frac{C_p}{C_p - C_v} = \frac{\gamma}{\gamma - 1} \]  
(2.16)

\[ dT = \frac{dQ_{HX}}{m_pC_p} + \frac{\gamma - 1}{\gamma} \frac{T}{p} dp \]  
(2.17)

where \( C_p \) is the constant-pressure specific heat of the gas and \( \gamma \) is:

\[ \gamma = \frac{C_p}{C_v} \]  
(2.18)
We know how the pressure changes, therefore \( p \) and \( dp \) are known. We need an expression for \( dQ_{HX} \) in terms of temperature or other parameters that we know. For \( dQ_{HX} \) different expressions can be proposed. One way is to use a steady-state Nusselt number for the particular geometry and the flow in question. Wakeland and Keolian (2004) employed another method. They proposed a relaxation time parameter \( \tau \) which accounts for the imperfection of the heat exchangers. By imperfection they meant that when a gas parcel comes in contact with a heat exchanger, its temperature does not instantly jump to the heat exchanger’s temperature. The reasoning behind introducing a relaxation time parameter is explained in the next paragraph. We also use the relaxation time parameter.

Imagine that a parcel comes in contact with a heat exchanger whose temperature is greater than that of the parcel at time \( t = t_0 \) that is: \( T_{HX} > T \) where \( T_{HX} \) represents the temperature of the heat exchanger and \( T \) is the temperature of the gas parcel. Heat transfers from the heat exchanger to the parcel and in the ideal case the parcel’s temperature becomes the temperature of the heat exchanger by the end of the time step at time \( t = t_0 + dt \). Therefore, the amount of heat transferred to the parcel in the ideal case can be calculated as:

\[
dQ_{HX,ideal} = m_p C_p (T_{HX} - T)
\]  

(2.19)

Note that we assumed that heat transfers during a constant-pressure process for the ideal case and that is why \( C_p \) appears in this equation. However, this never happens because \( dt \) is very small and the process is not an isobaric process. In reality, just a portion of heat expressed by this equation is transferred. The amount of heat transfer in the non-ideal case depends on the size of the time step \( dt \) and gas properties. Wakeland and Keolian (2004) introduced a time delay parameter \( \tau \) which made it possible to use a modified form of Eq. 2.19 to model the heat
transferred from the heat exchanger. Therefore, a new expression for heat transferred between 
\( t = t_0 \) and \( t = t_0 + dt \) can be proposed as follows:

\[
dQ_{HX} = m_p c_p (T_{HX} - T) \frac{dt}{\tau}
\]  

(2.20)

where \( \tau \) can be chosen according to Fig. 8 in (Wakeland and Keolian, 2004) as follows:

\[
\tau = \frac{0.9 y_0}{\pi \delta_k f}
\]  

(2.21)

which shows the correspondence between \( \frac{y_0}{\delta_k} \) and \( \tau f \) determined by measuring the overall effectiveness of the heat exchangers; \( y_0 \) is the half of the vertical space between two vertically adjacent heat exchangers, \( f \) is the frequency of oscillation; and \( \delta_k \) is the thermoacoustic thermal penetration thickness which is defined as follows,

\[
\delta_k = \frac{2K}{\sqrt{\rho c_p \omega}}
\]  

(2.22)

where \( K \) is the conductivity of the fluid and \( \omega \) is the angular frequency of the oscillation as mentioned before. Eq. 2.20 also works if the temperature of the heat exchanger is smaller that temperature of the gas parcel. In that case \( dQ_{HX} \) will be a negative number which means heat is going out of the system and is thermodynamically correct. Substituting Eq. 2.20 into Eq. 2.17 provides the expression for temperature change in a parcel of gas:

\[
dT = \frac{\gamma - 1}{\gamma} \frac{T}{p} dp + (T_{HX} - T) \frac{dt}{\tau}
\]  

(2.23)
Eq. 2.23 can be used in a simplified numerical model. In the simplified numerical model, we only look at the parcels that interact with both hot and cold heat exchangers. For every parcel, the implicit scheme to find the temperature change can be developed. Considering \(i\)-th parcel in the period between time step \(n\) and time step \(n + 1\):

\[
T_i^n - T_i^{n-1} = \frac{\gamma - 1}{\gamma} \frac{T_i^n}{p_i^n} (p_i^n - p_i^{n-1}) + (T_{HX} - T_i^n) \frac{dt}{\tau}
\]  

(2.24)

According to Eq. 2.4 pressure is only a function of time and is the same for all the parcels which means:

\[
p_i^n = p^n = p(t_n)
\]  

(2.25)

\[
p_i^{n-1} = p^{n-1} = p(t_{n-1})
\]

Finally, \(T_i^n\) can be found from Eq. 2.24:

\[
A_n T_i^n = A_{n-1} T_i^{n-1} + A_{HX} T_{HX}
\]

\[
A_{n-1} = 1
\]

\[
A_{HX} = \frac{dt}{\tau}
\]  

(2.26)

\[
A_n = A_{n-1} + A_{HX} - \frac{\gamma - 1}{\gamma} \frac{p^n - p^{n-1}}{p^n}
\]

It can be easily shown that for all time steps \(A_n > 0\) and the implicit scheme is completely stable for this problem. If the parcel is not in contact with any of the heat exchangers \(A_{HX} = 0\).
The algorithm of the simplified model is as follows: The pressure, position, and velocity of every parcel at any time step are given by equations 2.4, 2.6, and 2.7. The initial temperature for parcels could be the temperature distribution when the fluid is at rest and not oscillating. Using equations 2.26, the temperature of every parcel can be found at every time step until the acoustic cycle is over. By the end of the acoustic cycle, the last temperature calculated for every parcel will be used as the new initial condition at the beginning of the next cycle. This process is performed over and over again until for every parcel a stable periodic temperature curve is achieved. As soon as temperature is determined for every parcel at every time step throughout one acoustic cycle, by using Eq. 2.20 and integrating over all the parcels, the time-average heat transfer rates during one cycle can be determined. If at n-th time step I parcels are in contact with the hot heat exchanger and J parcels are in contact with the cold heat exchangers for every single parcel we can write:

\[
dQ_{HHX,i}^n = m_p C_p (T_{HHX} - T_i^n) \Delta t \frac{\Delta t}{\tau}
\]

(2.27)

\[
dQ_{CHX,j}^n = m_p C_p (T_{CHX} - T_j^n) \Delta t \frac{\Delta t}{\tau}
\]

then the heat transfer rate can be found for n-th time step as follows:

\[
\dot{Q}_{HHX}^n = \frac{1}{\Delta t} \sum_{i=1}^{I} dQ_{HHX,i}^n
\]

(2.28)

\[
\dot{Q}_{CHX}^n = \frac{1}{\Delta t} \sum_{j=1}^{J} dQ_{CHX,j}^n
\]
finally, the average heat transfer rates for hot and cold heat exchangers can be calculated respectively:

\[
\dot{Q}_{HHX} = f \int_{t=0}^{t=\frac{1}{T}} \dot{Q}_{HHX}^n dt
\]

(2.29)

\[
\dot{Q}_{CHX} = f \int_{t=0}^{t=\frac{1}{T}} \dot{Q}_{CHX}^n dt
\]

To calculate efficiency, a model for viscous losses is required. To account for viscous loss, we use the quasi-steady expression for power loss in channels with smooth surfaces. The power loss is determined for half of a channel with two heat exchangers:

\[
\dot{W}_{Loss} = \dot{W}_{Loss,HHX} + \dot{W}_{Loss,CHX}
\]

\[
\dot{W}_{Loss,HHX} = -\Delta P_{Loss,HHX} \bar{U}_{HHX} y_0
\]

(2.30)

\[
\dot{W}_{Loss,CHX} = -\Delta P_{Loss,CHX} \bar{U}_{CHX} y_0
\]

where \( \bar{U}_{HHX} \) is the time-dependent spatially averaged velocity amplitude of all the parcels come in contact with the hot heat exchanger, \( \bar{U}_{CHX} \) is the time-dependent spatially averaged velocity amplitude of all the parcels come in contact with the cold heat exchanger, and \( \Delta P_{Loss} \) for each heat exchanger can be found as follows:

\[
\Delta P_{Loss,HHX} = \frac{1}{2} F_{HHX} \frac{L_{HHX}}{D_h} \rho_m \frac{\bar{U}_{HHX}^2}{2}
\]

(2.31)

\[
\Delta P_{Loss,CHX} = \frac{1}{2} F_{CHX} \frac{L_{CHX}}{D_h} \rho_m \frac{\bar{U}_{CHX}^2}{2}
\]
The coefficient $\frac{1}{2}$ shows that we are considering only half of the channel in the simplified numerical model. $L_{HHX}$ and $L_{CHX}$ are respectively the lengths of hot and cold heat exchangers, $F$ is the friction coefficient for smooth surfaces (Heat exchangers in this case) in channels which is a function of the Reynolds number:

$$F_{HHX} = \frac{64}{Re_{D,HHX}}$$  
$$F_{CHX} = \frac{64}{Re_{D.CHX}}$$  

Reynolds number for the flow between heat exchanger plates can be expressed by:

$$Re_{D,HHX} = \frac{\rho_m \bar{U}_{HHX} D_h}{\mu_m}$$  
$$Re_{D,CHX} = \frac{\rho_m \bar{U}_{CHX} D_h}{\mu_m}$$  

$D_h$ (which is equal to $4y_0$), $\rho_m$, $\mu_m$ are the hydraulic diameter, the mean density and the mean viscosity of the gas, respectively.

After calculating the viscous losses and heat transfer rates, the second-law efficiency of the system as an engine or a heat pump can be calculated. The ideal (non-viscous) power input/output can be found from the first law of thermodynamics for a cycle:
\[ \dot{W}_{NV} = \dot{Q}_{HHX} + \dot{Q}_{CHX} \]  

(2.34)

\( \dot{W}_{NV} \) represents non-viscous power. Also, the net generated or consumed acoustic power can be determined as follows:

\[ \dot{W}_{net} = \dot{W}_{NV} + \dot{W}_{Loss} \]  

(2.35)

and \( \dot{W}_{Loss} \) can be determined using equations 2.30 to 2.33. For a heat pump \( \dot{Q}_{HHX} \) is negative, \( \dot{Q}_{CHX} \) is positive, and as a result \( \dot{W}_{net} \) is negative. For an engine, \( \dot{W}_{net} \) and \( \dot{Q}_{HHX} \) are positive and \( \dot{Q}_{CHX} \) is negative. \( \dot{W}_{Loss} \) is a negative number for both cases which is added to the ideal power resulting in smaller useful power for an engine and greater consumed power for a heat pump. The coefficient of performance for a heat pump and the thermal efficiency of an engine can be defined as follows:

\[ COP = \frac{\dot{Q}_{HHX}}{\dot{W}_{net}} \]  

(2.36)

\[ \eta_{th} = \frac{\dot{W}_{net}}{\dot{Q}_{HHX}} \]  

(2.37)

Finally, the second law efficiency for the heat pump and the engine can be found:

\[ \eta_{II(Heat\ Pump)} = \frac{COP}{COP_{C}} \]  

(2.38)

\[ \eta_{II(Engine)} = \frac{\eta_{th}}{\eta_{C}} \]  

(2.39)
where \( \text{COP}_C \) and \( \eta_C \) are Carnot coefficient of performance and Carnot thermal efficiency respectively:

\[
\text{COP}_C = \frac{1}{1 - \frac{T_C}{T_H}} \quad (2.40)
\]

\[
\eta_C = 1 - \frac{T_C}{T_H} \quad (2.41)
\]

For our problem, \( T_C = T_{CHX} \) and \( T_H = T_{HHX} \). We are more interested in the second-law efficiency because it gives us a better understanding of the performance of each system in reality since our setups are measured against the most thermodynamically optimum setups possible. Also, studying the second law efficiency is more sensible when low-grade heat sources are used.

**CFD Model**

Important aspects of the CFD cases in Fluent are discussed in this section. The geometry is half of the setup shown in Fig. 2.2. The bottom line is a symmetry line, which divides the channel into 2 sections and as a result only half of a channel is modeled to minimize the computational effort. A rectangular mesh was used with \( \Delta x = (5 \times 10^{-5})\lambda \) and \( \Delta y = \frac{\delta_k}{35} \) where \( \lambda \) is the acoustic wavelength and \( \delta_k \) can be determined from Eq. 1.7
Figure 2.2 No-stack system geometry. Top sub-figure: studied parameters in simplified numerical modeling as well as in CFD modeling. One of gas parcels is shown as a thin vertical bar. Double arrow shows acoustic motion of the parcel. Bottom sub-figure: boundary conditions for CFD modeling.

The 2nd-order implicit unsteady formulation was employed to advance in time. A laminar viscous model was used because the Reynolds number is sufficiently small. The working gas, helium, is modeled as an ideal gas with constant isobaric specific heat. Conductivity and viscosity of the fluid are given to the code as functions of temperature. The gas (helium) properties entered into the software from Wallard et al. (2008) are:

\[ C_p = 5193 \frac{J}{Kg \cdot K} \]  
(2.42)

\[ k = (0.25672 \times 10^{-2})T_{cell}^{0.716} \frac{W}{m \cdot K} \]  
(2.43)

\[ \mu = (0.412 \times 10^{-6})T_{cell}^{0.68014} \frac{Kg}{m \cdot s} \]  
(2.44)

\[ M = 4.0026 \frac{Kg}{Kmol} \]  
(2.45)

where \( M \) is the molecular mass. \( T_{cell} \) is the absolute temperature of a cell in the mesh. Therefore, viscosity and conductivity of every cell are functions of the most recent temperature calculated for the cell and they vary from cell to cell. This provides a more accurate approximation of these
two properties. Boundary conditions are shown in Fig. 2.2. The heat exchangers are modeled as stationary walls with constant temperatures. The pressure and temperature on the right boundary (pressure inlet) change adiabatically and periodically according to the following equations:

\[
p(t) = p_m + P_A \sin \omega t
\]  
\[
T(t) = T_c + \frac{\gamma - 1}{\gamma} \frac{T_c}{p_m} P_A \sin \omega t
\]

where \(T_c\) and \(p_m\) are cold heat exchanger temperature and mean pressure, respectively. Equation 2.47 shows the adiabatic expression for the temperature of an ideal gas with pressure fluctuations. Equation 2.47 can be derived from Eq. 2.17 by setting \(dQ_{HX} = 0\) in that equation. For our cases, \(p_m = 1\) MPa, \(P_A = DR \cdot p_m\). \(DR\) is the drive ratio which varies from 20% to 50% for different cases. This type of oscillating temperature and pressure is given to a FLUENT code through user-defined functions (UDF). General solution controls for our cases are given as follows. Pressure and velocity coupling is chosen to be PISO, and the discretization scheme is the second order upwind. Convergence criteria (residuals) are \(1 \times 10^{-4}\) for continuity, \(x\)-velocity, and \(y\)-velocity equations and \(1 \times 10^{-6}\) for the energy equation. For initialization (at \(t=0\)), pressure is \(p_m = 1\) MPa and velocities are zero. \(\Delta t\) must be chosen small enough to achieve desired accuracy and avoid numerical diffusion. In this work \(\Delta t\) is chosen as follows:

\[
\Delta t = \frac{1}{Nf}
\]

where \(f\) is the frequency of oscillation. \(N\) is the number of time steps over one cycle which varies from \(N = 100\) to \(N = 200\). We mostly used \(N = 100\) in this work.
The initial temperature, which is given by another UDF function, is the steady energy solution for the system when the fluid is at rest. FLUENT solves the general mass, momentum, and energy conservation equations for this flow. These equations are:

Mass conservation: \[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{V}) = 0 \quad (2.49)
\]

Momentum equation: \[
\rho \left[ \frac{\partial \vec{V}}{\partial t} + (\vec{V} \cdot \nabla)\vec{V} \right] = -\nabla p + \nabla \cdot \sigma \quad (2.50)
\]

Energy equation: \[
\frac{\partial}{\partial t} \left( \rho e + \frac{1}{2} \rho V^2 \right) = -\nabla \cdot \left[ \left( \rho h + \frac{1}{2} \rho V^2 \right) \vec{V} - K \nabla T - \nabla \cdot \sigma \right] \quad (2.51)
\]

Where \( \vec{V} \) is the velocity vector, \( \sigma \) is the viscous stress tensor which has 9 components, \( e \) is the internal energy, and \( h \) is the enthalpy of the fluid. \( \sigma \) is slightly different for compressible flow and that’s why we did not write equation 2.49 in the form of Navier-Stokes equations. The stress tensor for compressible flow is:

\[
\sigma_{ij} = -p \delta_{ij} + \mu \left[ \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3} \delta_{ij} \nabla \cdot \vec{V} \right] \quad (2.52)
\]

After running every case for 40 to 50 thermoacoustic cycles, fluid properties seem to change periodically over every cycle. Input/output time-average power at the right side of the flow domain (pressure inlet) can be determined using the following expression:

\[
\dot{W}_{\text{average}} = f \int_0^1 \dot{W}(t) \, dt \quad (2.53)
\]
\[ \dot{W}(t) = \int_{A_{inlet}} p'(t)u'(t)dA_{inlet} \quad (2.54) \]

where \( A_{inlet} \) in Eq. 2.54 is the area of the pressure inlet in our model (Fig. 2.2). Using an integral monitor on the pressure inlet enables us to calculate \( \dot{W}(t) \) for every time step. Numerical form of Eq. 2.53 for \( N \) time steps over one acoustic cycle is:

\[ \dot{W}_{average} = f \sum_{n=1}^{N} \dot{W}_n(t) \Delta t \quad (2.55) \]

Using Eq. 2.48:

\[ \dot{W}_{average} = \frac{1}{N} \sum_{n=1}^{N} \dot{W}_n(t) \quad (2.56) \]

Heat transfer rates from heat exchangers can be calculated using the following equations:

\[ \dot{Q}_{HHX, average} = f \int_{0}^{1} \dot{Q}_{HHX}(t)dt \quad (2.57) \]

\[ \dot{Q}_{HHX}(t) = \int_{A_{HHX}} q''_{HHX}(t)dA_{HHX} \quad (2.58) \]

where \( A_{HHX} \) in Eq. 2.58 is the area of the hot heat exchanger in our model (Fig. 2.2). Using an integral monitor on the hot heat exchanger enables us to calculate \( \dot{Q}_{HHX}(t) \) for every time step. \( q''_{HHX}(t) \) is total surface heat flux which can be found among wall fluxes’ monitors on FLUENT. Numerical form of Eq. 2.57 for \( N \) time steps over one acoustic cycle is:
\[ \dot{Q}_{\text{HHX,average}} = f \sum_{n=1}^{N} \dot{Q}_{\text{HHX},n}(t) \Delta t \]  \hspace{1cm} (2.59)

Using Eq. 2.48:
\[ \dot{Q}_{\text{HHX,average}} = \frac{1}{N} \sum_{n=1}^{N} \dot{Q}_{\text{HHX},n}(t) \]  \hspace{1cm} (2.60)

In the same manner, heat transfer rate from the cold heat exchanger can be determined:

\[ \dot{Q}_{\text{CHX,average}} = f \int_{0}^{1} \dot{Q}_{\text{CHX}}(t) dt \]  \hspace{1cm} (2.61)

\[ \dot{Q}_{\text{CHX}}(t) = \int_{A_{\text{CHX}}} q''_{\text{CHX}}(t) dA_{\text{CHX}} \]  \hspace{1cm} (2.62)

\[ \dot{Q}_{\text{CHX,average}} = f \sum_{n=1}^{N} \dot{Q}_{\text{CHX},n}(t) \Delta t \]  \hspace{1cm} (2.63)

Using Eq. 2.48:
\[ \dot{Q}_{\text{CHX,average}} = \frac{1}{N} \sum_{n=1}^{N} \dot{Q}_{\text{CHX},n}(t) \]  \hspace{1cm} (2.64)
CHAPTER 3

RESULTS FROM NO-STACK MODELS
Results From No-Stack Models

The main objective of this work is to study the thermoacoustic performance of intermittent stacks numerically. One suggested pattern in the previous chapter was the no-stack system, consisting of two plates that serve as heat exchangers. We employ two methods to study this system: (1) computationally efficient Simplified Numerical Model (SNM), and (2) more detailed Computational Fluid Dynamics (CFD) which will presumably provide more accurate results. However, before modeling the whole case numerically, there is one area that we need to gain confidence in: The CFD code must be reliable when modeling oscillating flow and thermoacoustic effects. Therefore, some validation is a necessary step before modeling a new system. During the validation process, we also learn how to choose different numerical parameters such as mesh size and time step.

Validation

The goal of validation is to gain more confidence in the results, ensuring the reliability of the code in modeling oscillating flow and thermoacoustic effects. For this purpose, we look at a problem for which there is an exact theoretical solution. The setup is a stack plate inside a resonator (Fig. 3.1). The mean pressure for this case is \( p_m = 1 \text{ MPa} \) and \( DR = 2\% \) at the pressure inlet on the right edge with an oscillation frequency of \( f = 100 \text{ Hz} \). There is no temperature gradient along the stack and mean temperature is \( T_m = 300 \text{ K} \) throughout the resonator including the stack. At low amplitudes of acoustic oscillations, some flow properties, such as pressure and velocity, vary nearly sinusoidally in time. But their amplitudes and phase shifts are dependent on spatial coordinates. Therefore, a meaningful comparison would be the
one between the analytically predicted amplitudes and phase shifts and those values of numerical results.

According to Swift (1988), oscillating temperature and velocity can be expressed as follows:

\[
T(x, y, t) = T_m(x) + T'(x, y, t) = T_m(x) + \text{Re}(T_1(x, y)e^{i \omega t}) \tag{3.1}
\]

\[
u(x, y, t) = \nu'(x, y, t) = \text{Re}(\nu_1(x, y)e^{i \omega t}) \tag{3.2}
\]

where \(T'\) and \(\nu'\) are the real oscillating parts of temperature and velocity which are expressed in terms of complex \(T_1\) and \(\nu_1\) which are only functions of spatial coordinates not time. A comparison of normalized velocity amplitudes \(|\nu_1|/\frac{\rho A}{\rho_m \cdot \alpha}\) and normalized temperature amplitudes \(|T_1|/\frac{\rho A}{\rho_m \cdot c_p}\) obtained with the CFD code and theoretical results of Swift (2002) at the specified section, right at the middle of the stack plate, are given in Fig. 3.2. In the normalized amplitudes, \(\rho_m\) and \(\alpha\) are respectively the mean density and the speed sound of the working fluid and \(P_A\) is the pressure amplitude.
To study the phase shift we chose an integral which is dependent on both amplitudes and phase shift between temperature and velocity at the cross-sectional area shown in Fig. 3.1. The following time average integral is an important part of thermoacoustic enthalpy flux along the channel (Swift, 2002):

$$ f \int_0^1 T' u' dt = \frac{1}{2} |T_1||u_1| \cos \varphi_{Tu} \tag{3.3} $$

where $\varphi_{Tu}$ is the phase shift between temperature and velocity and $f$ is the frequency. It is apparent that this parameter is dependent on both amplitudes and phase shift. Fig. 3.3 shows the calculated integral based on CFD and theoretical results. The FLUENT model provides an excellent agreement with the linear acoustic theory at low drive ratios.
Figure 3.3 Time-average integral of the product of oscillating temperature and velocity at the cross-section in the middle of the stack plate.

Results from studying specific cases

No-stack, two-plate configurations of heat pumps and prime movers are modeled here with the following goals: (1) to determine the dependence of system performance on system parameters, (2) to check the difference between simplified and complete modeling, and (3) to find optimum operational states. Mean pressure was $p_m = 1$ MPa for all cases. Heat exchanger temperatures were chosen to be $T_C = 300$ K and $T_H = 340$ K for heat pumps and $T_C = 300$ K and $T_H = 500$ K for prime movers. For all no-stack cases $y_0 = 0.1$ mm except for the validation case for which $y_0 = 0.2$ mm (Fig. 2.2). $y_0$ is chosen to be a little smaller than but very close to the thermal penetration depth $\delta_k$ for all cases. Also, heat pumps are working with a frequency of $f = 320$ Hz and prime movers with a frequency of $f = 250$ Hz. Variable parameters included $x_n$, $L_{hx}$, $x_{hx}$ (Fig. 2.2) and $DR$, with $x_n$ being the distance from a point equidistant between the heat exchangers to the wall, $L_{hx}$ being the length of heat exchangers, $x_{hx}$ being half of the distance between the heat exchangers, and $DR$ being the drive ratio. Calculated metrics for each case were the efficiency, heat transfer rates, and acoustic power (produced or consumed). Using
the simplified numerical model, promising ranges with high system performance for each of the above parameters were found. These ranges are \(20\% < DR < 50\%\), \(0.5\, \text{mm} < L_{hx} < 2\, \text{mm}\), \(12\, \text{mm} < x_n < 28\, \text{mm}\) and \(1.3\, \text{mm} < x_{hx} < 3\, \text{mm}\). By changing parameters within these ranges, one can expect to find the optimum values for each parameter where the second law efficiency becomes maximum. Besides the second law efficiency, the generated power, \(\dot{W}\), is also important in the case of prime movers as well as heat transfer rates, \(\dot{Q}_H\), in the case of heat pumps. The reason is that a prime mover might demonstrate high second law efficiency but the amount of generated power might not be great enough for a particular application. Also, a thermodynamically efficient refrigerator might pump an insufficient amount of heat to the hot heat exchanger over one thermoacoustic period. Power density or heat transfer rates are important because they define the size of a system for a particular application.

In Fig. 3.4 the second law efficiency, power and heat transfer rate from the hot heat exchanger are shown for the variable \(x_n\). Trends are similar in both CFD and simplified models. For both the prime mover and heat pump with a constant \(DR\), the second law efficiency reaches a maximum at a particular \(x_n\). With increasing \(x_n\), the acoustic pressure amplitude between the heat exchangers decreases at a smaller rate in comparison with an increase of the acoustic velocity amplitude. Since the generated acoustic power is proportional to both pressure and velocity amplitudes (Swift, 2002), the acoustic power increases with \(x_n\). The heat transfer rate also depends on the fluctuating temperature amplitude, which increases with pressure amplitude. This results in the extremum point for \(\dot{Q}_H\).
Results from varying $x_{hx}$ (half of the distance between the heat exchangers) are shown in Fig. 3.5. The prime mover curves show a similar trend by both models. The second law efficiency reaches a maximum; and the optimal $x_{hx}$ from both models are close. Also, power follows the same descending trend, and values are comparable. To explain this, the displacement amplitude of parcels can be considered. As $x_{hx}$ increases, fewer parcels come in contact with both heat exchangers resulting in lower heat being pumped from the hot heat exchanger to the cold one. This makes the absolute values of $\dot{Q}_H$, $\dot{Q}_C$ and $\dot{W}$ smaller. For the heat pump, results are different. Absolute values of $\dot{Q}_H$ actually increase with $x_{hx}$, which is likely due to lower temperature difference than for the prime mover case.
Figure 3.5 Effect of $x_{hx}$ on the performance of no-stack prime mover and heat pump. Curves, simplified numerical modeling (SNM); points, CFD results. Left: Prime mover, $f = 250$ Hz, $DR = 0.3$, $L_{hx} = 1.5$ mm, $x_n = 16$ mm. Right: Heat pump, $f = 320$ Hz, $DR = 0.35$, $L_{hx} = 1.3$ mm, $x_n = 22$ mm. $\dot{Q}_H$ and $W$ are given for half-channel.

Figures 3.6 and 3.7 show the effect of variation in $L_{hx}$ and $DR$ on the performance of no-stack prime mover and heat pump. In these cases greater difference between the results from both models were noticed. The reason lies in the fact that these two parameters are directly related to the friction loss expression in Eqs. 2.30 and 2.31. In the simplified model, power loss is proportional to $L_{hx}$ (heat exchanger length) and $U^3$. $U$ (average parcels velocity) is determined by $DR$. Larger $DR$ results in higher $U$. Since viscous dissipation is not calculated accurately in the simplified model, there is greater discrepancy between simplified model and CFD results in $L_{hx}$ and $DR$ curves. Increasing $L_{hx}$ (Fig. 3.6) has the same effect on both prime movers and heat pumps. Longer heat exchanger means that more parcels can come in contact with both heat exchangers, which leads to greater absolute heat transfer rates and power. However, power loss also increases with $L_{hx}$ and efficiency decreases. In both configurations, with increasing $DR$, the displacement amplitude also increases, resulting in more parcels coming in contact with both heat exchangers. Therefore, absolute values of heat transfer rates and power will go up which is predicted by both models in Fig. 3.7.
Figure 3.6 Effect of $L_{hx}$ on the performance of no-stack prime mover and heat pump. Curves, simplified numerical modeling (SNM); points, CFD results. Left: Prime mover, $f = 250$ Hz, $DR = 0.3$, $x_{hx} = 2.3$ mm, $x_n = 16$ mm. Right: Heat pump, $f = 320$ Hz, $DR = 0.35$, $x_{hx} = 2.3$ mm, $x_n = 22$ mm. $\dot{Q}_H$ and $W$ are given for half-channel.

Figure 3.7 Effect of $DR$ on the performance of no-stack prime mover and heat pump. Curves, simplified numerical modeling (SNM); points, CFD results. Left: Prime mover, $f = 250$ Hz, $x_{hx} = 2.3$ mm, $x_n = 16$ mm, $L_{hx} = 1.5$ mm. Right: Heat pump, $f = 320$ Hz, $x_{hx} = 2.3$ mm, $x_n = 22$ mm, $L_{hx} = 1.3$ mm. $\dot{Q}_H$ and $W$ are given for half-channel.

It is apparent from the curves that there are quantitative differences between results from the two models, especially in the case of heat pumps. There are several reasons for these discrepancies: (1) The quasi-steady, fully developed internal flow model from the previous chapter was employed for calculating viscous dissipation in the simplified numerical model, while it is not entirely realistic as the heat exchangers do not extend throughout the resonator. (2)
In the CFD modeling viscous heating is taken into account but the simplified numerical model does not include that. (3) The simplified model assumes that there is only one isothermal gas parcel at a time at any cross section (Fig. 3). This implies that in the simplified model there is no temperature gradient in y direction which affects the accuracy of results. (4) The simplified approach does not account for the conduction heat transfer between the gas parcels while conductivity is defined as a function of temperature for every cell in the CFD model.

In general, the simplified numerical model saves a lot of time and effort to find the optimum geometries but lacks accuracy in some cases. Also, there is a better agreement between the two models in the case of prime movers. This is because larger temperature difference between the heat exchangers (200 K) leads to greater heat transfer rates and power, which outweigh the calculated power loss. Based on the results discussed above, efficient heat pump and prime mover configurations are depicted in Fig. 3.8. These systems demonstrated the best second law efficiency and higher $\dot{W}_{\text{net}}$ in the case of prime movers as well as higher absolute $\dot{Q}_H$ in the case of heat pumps. The thermal efficiency of the optimum prime mover is 16%, with a second law efficiency of 40%, and the coefficient of performance of the optimum heat pump is 4.68, with a second law efficiency of 55%. For all cases thermal efficiencies and coefficients of performance can be calculated using Eqs. 2.38, 2.39, and presented results.
The direct applicability of the obtained results here can be questioned because the real-world heat exchangers have plates with a finite thickness while our models considered the systems with zero-thickness plates. To explore the effect of the finite thickness, the optimum heat pump (Fig. 3.8, bottom) was modeled with non-zero thickness heat exchanger plates. To characterize plate thickness, a new parameter is introduced:

$$\Phi = \frac{t_{hx}}{y_0}$$  \hspace{1cm} (3.4)

where $t_{hx}$ is heat exchanger thickness (Fig. 3.9) and $\Phi$ is called porosity which varies between 0 and 1.

The optimum heat pump (Fig. 3.8) with porosities of $\Phi = 0, 0.25, 0.5, 0.60$ was studied and the 2nd law efficiency and heat transfer rates obtained from CFD modeling are presented in Fig. 3.10. Since for non-zero-thickness plates the mesh in the vicinity of heat exchangers has to be
denser in order to resolve vortices that might be formed around the heat exchanger edges, these cases take more CPU time. It is clear from Fig. 3.10 that the values of the second law efficiency and time average heat transfer rate for $\Phi = 0$ and $\Phi = 0.25$ in Fig. 3.10 are almost the same. It can be concluded that for CFD simulation, ignoring heat exchanger thickness is a reasonable approximation in low-porosity configurations. Using this approximation, coarser mesh can be applied which saves hours of computational effort. For example, running a case with a porosity of 25% requires at least 4 times more CPU time than for an equivalent zero-thickness case. As $\Phi$ increases, heat exchangers obstruct the flow, which leads to substantial reduction of efficiency.

![Figure 3.10](image)

**Figure 3.10** Effect of porosity on the performance of no-stack heat pump.

One of motivations for this study was to explore if thermoacoustic no-stack systems can be more efficient than conventional systems with the stack. Using linear acoustic theory, some stack systems with the same dimensions of that of no-stack ones were also studied. The same temperatures, working gas, $y_0$, $p_m$, and $DR$ were applied. The outer ends of the stack system were positioned at the same location where the outer edges of the heat exchangers in the no-stack system had been placed. Similar second-law efficiencies were achieved in the stack systems but the advantage of the no-stack system was in greater values for power and heat transfer rates for the same second law efficiency. However, since the stack case was not separately optimized the comparison was not entirely exact. In addition, the results from linear acoustic theory for drive ratio bigger than 3% are not reliable (Swift, 2002); and this study is focused on high $DR$.  

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The agreement between the simplified numerical model and results from FLUENT provided motivation to take a step further and look at no stack systems in a more general way. As we see, there are parameters such as $L_{hx}$, $x_{hx}$, and $DR$ that are almost the same for both engines and heat pumps and there are parameters such as temperature difference, $\Delta T = T_H - T_C$, and $x_n$ that can determine the application of a system as a heat pump or an engine. That is by changing the temperature difference between the heat exchangers, and engine can be converted to a heat pump or vice versa. The purpose of simplified numerical model was to study a broader range of parameters in a shorter period of time. Therefore, we modeled a broader range of the above parameters. To study $\Delta T = T_H - T_C$, a suitable approach would be to keep one temperature constant and change the other one from a minimum value to a maximum value. Since the standard ambient temperature is always available it is reasonable to simulate a system with $T_C = 300$ K and variable $T_H$ or a system with $T_H = 300$ K and variable $T_C$. We used the same $L_{hx} = 1.3$ mm, $x_{hx} = 2.3$ mm, $f = 320$ Hz, and $DR = 35\%$ from our previous studies. However, to keep the system smaller in size we picked $x_n = 15$ mm.

Figure 3.11 shows the variation of the second law efficiency and heat transfer rates of the system described in the previous paragraph for a constant $T_C = 300$ K and a range of $T_H$ from $T_H = 305$ K to $T_H = 600$ K. As we see, there are two areas in this figure. One area shows a range of $T_H$ for which the system works as a heat pump and the other area corresponds to a range of $T_H$ that causes the system to perform as an engine. For each area, there is a peak that represents the maximum efficiency. The simplified numerical model predicts that the maximum second law efficiency for this system can be greater than 60\% for both cases. One can pose this question that why is the engine efficiency smaller at $T_H = 600$ K than $T_H = 425$ K? The reason lies in the definition of the second law efficiency for an engine. $\eta_{II}$ for an engine is defined as follows:
\[ \eta_H = \frac{1 - \frac{\dot{Q}_C}{\dot{Q}_H}}{1 - \frac{T_C}{T_H}} \]  

(3.5)

The denominator of this fraction increases as \( T_H \) increases and \( T_C \) is kept constant while in the numerator, the absolute values of \( \dot{Q}_C \) and \( \dot{Q}_H \) will both increase with \( \Delta T = T_H - T_C \). Therefore, the denominator is increasing with a greater rate than the numerator and the maximum of this fraction does not occur at higher \( \Delta T \).

![Figure 3.11 The conversion of a no stack thermoacoustic system from a heat pump to an engine by changing \( T_H \)](image)

Also, for a small range of \( T_H \) the system seems to be dissipative and inefficient or acoustic power is neither produced nor absorbed. This is where the system is neither a heat pump nor an engine.
The fact that changing $\Delta T = T_H - T_C$ can change the function of a thermoacoustic system from a heat pump to an engine and vice versa can be explained by a mathematical parameter, critical temperature gradient, that exists in linear thermoacoustic theory. According to Swift (2002), the ideal critical temperature gradient for a thermoacoustic system is defined as:

$$\left| \nabla T_{cr, id} \right| = \frac{|p_1|}{\rho_m C_p} = \frac{|p_1 \omega|}{\rho_m C_p |u_1|}$$ \hspace{1cm} (3.6)

where $|p_1|$ and $|u_1|$ are the absolute values of complex pressure and velocity that were introduced before. $p_1$ and $u_1$ can be derived from Eqs. 1.2 and 1.4 as follows:

$$p_1(x) = P_A \cos kx$$ \hspace{1cm} (3.7)

$$u_1(x) = \frac{P_A}{i \rho_m a} \sin kx$$ \hspace{1cm} (3.8)

As mentioned earlier, in this chapter, we are looking at a small portion of the resonator which is much smaller than the wavelength. Since $k = \frac{2\pi}{\lambda}$ and $x / \lambda \sim 0$:

$$\cos kx = \cos \frac{2\pi x}{\lambda} \sim 1$$ \hspace{1cm} (3.9)

$$|p_1(x)| = |P_A \cos kx| \sim P_A$$ \hspace{1cm} (3.10)

Also:

$$|u_1(x)| = \left| \frac{P_A}{\rho_m a} \sin kx \right|$$ \hspace{1cm} (3.11)

Therefore for our simplified numerical model:
This equation that is derived for non-viscous flow suggests that when the temperature gradient along a thermoacoustic system is equal to $\nabla T_{cr,ld}$, acoustic power is neither produced nor absorbed. In other words, it refers to the point where both $\dot{Q}_H$ and $\dot{Q}_C$ are zero. Also that is the transition point from a heat pump to an engine where efficiency is zero. For our study, Eq. 3.12 can be used to predict the transition temperature while changing $T_H$ or $T_C$ or to predict the transition $x_n$ when that parameter is studied. Such predictions will be shown later in Fig. 3.14.

In another study, we kept $T_H = 300$ K constant and changed $T_C$ from $T_C = 200$ K to $T_C = 295$ K. Other parameters were the same as the previous study. Figure 3.12 shows the variation of second law efficiency and heat transfer rates as $T_C$ varies.
Figure 3.12 The conversion of a no stack thermoacoustic system from an engine to a heat pump by changing $T_c$.

For this system, we also observe two areas, two peaks of second law efficiency and again the model predicts that the setup can perform as an efficient heat pump or an efficient engine with second law efficiencies over 60%. Again, by increasing $T_c$, the application of the system changes from an engine to a heat pump and the transition range of $T_c$ is obvious which represents an inefficient system that does not generate or consume acoustic power. The same explanation based the critical temperature gradient can also be applied to this figure.

Variation in $x_n$, was also studied for a system with $T_c = 300 \text{ K}$, $T_H = 400 \text{ K}$, $f = 320 \text{ Hz}$, $x_{hx} = 2.3 \text{ mm}$, and $L_{hx} = 1.3 \text{ mm}$. $y_0 = 0.1 \text{ mm}$ for this case and all the cases so far
except for the validation case for which \( y_0 = 0.2 \text{ mm} \). Figure 3.13 shows the variation of the second law efficiency and heat transfer rates as \( x_n \) varies.

Figure 3.13 The conversion of a no stack thermoacoustic system from a heat pump to an engine by changing \( x_n \)

Eq. 3.12 shows that the critical temperature is also dependent on \( x_n \). The best position to calculate the temperature gradient at in the resonator will be a point right between the heat exchangers equidistant from both heat exchangers. This point is \( x_n \). Therefore, we can rewrite the ideal critical temperature as follows:

\[

\nabla T_{cr, id} = \frac{a \omega}{C_p \sin k x_n} \tag{3.13}

\]
This equation can again be used to explain the appearance of two different areas in Fig. 3.13. \( \nabla T_{cr} \) corresponds to a point where \( \dot{Q}_H = \dot{Q}_C = 0 \) and in this figure it is \( x_n = 31.35 \text{ mm} \). This equation can provide a rough estimation of the particular \( x_n \) at which the transition from a heat pump to an engine occurs. Predictions by Eq. 3.13 for all last 3 studies are shown in Fig. 3.14.

<table>
<thead>
<tr>
<th>Studied parameter</th>
<th>Predicted transition point using ( \nabla T_{cr,id} )</th>
<th>Calculated transition point using SNM</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_C )</td>
<td>( T_C = 246 \text{ K} )</td>
<td>( T_C = 232 \text{ K} )</td>
</tr>
<tr>
<td>( T_H )</td>
<td>( T_H = 370 \text{ K} )</td>
<td>( T_H = 387 \text{ K} )</td>
</tr>
<tr>
<td>( x_n )</td>
<td>( x_n = 10.8 \text{ mm} )</td>
<td>( x_n = 31.35 \text{ mm} )</td>
</tr>
</tbody>
</table>

Figure 3.14 A comparison between the predictions from Eq. 3.12 and calculated transition points from SNM (Simplified Numerical Model)

The ideal critical temperature gradient is different from the temperature gradient that we calculated for the transition point in our simplified model. There are two major reasons: (1) Our model accounts for viscous effects. As we see the discrepancy between the predicted transition \( x_n \) and calculated value from SNM is great since viscous effects grow by increasing \( x_n \) leading to increasing acoustic velocity. (2) Linear acoustic theory was developed for small drive ratios usually \( DR < 3\% \) while for no stack systems we deal with \( DR > 30\% \). Generally, Eq. 3.13 provides a qualitative rough estimation of how the application of the system will change when we change one parameter.

It can be concluded that by changing the key parameters, every no stack thermoacoustic system can act as a thermoacoustic heat pump or engine. Using the simplified model one can choose the desirable parameters (size, temperature difference) and find the efficient points. The next step is to verify this model again by running CFD cases in the vicinity of optimum points.
To ensure the validity of the simplified model again, we look at the optimum ranges from our previous study of $T_C$ and $T_H$ corresponding to Figs. 3.11 and 3.12. Figures 3.15 and 3.16 show the results from simplified model and CFD simulations around the maximum efficiency points suggested by our simplified model. Figure 3.15 shows the results from studying $T_H$ and Fig. 3.16 shows the results from studying $T_C$.

![Graphs showing variation of second law efficiency with $T_H$ based on CFD results and SNM results](image)

Figure 3.15 Variation of second law efficiency with $T_H$ based on CFD results and SNM results; up: the system is acting as an engine; down: the system is acting as a heat pump.

Generally, there is a good agreement between CFD results and SNM results. However, it seems that in both figures the two curves are following the same trend in the case of engines while the curves do not look alike qualitatively in case of heat pumps. That is, for engines the
maximum $\eta_{II}$ occurs at the same points in both figures while in the case of heat pumps the maximum $\eta_{II}$ occurs at different points although the numbers are close. This is explicable because the magnitude of $\dot{Q}_H$ and $\dot{Q}_C$ are bigger for engines due to bigger temperature difference. Greater heat transfer rates mean greater generated power which diminishes the lack of accuracy in our model. This lack of accuracy, as discussed before, is due to the viscous model that we used. FLUENT on the other hand, solves full momentum and energy equations which also account for viscous heating and viscous losses more accurately.

![Graph](image)

Figure 3.16 Variation of second law efficiency with $T_c$ based on CFD results and SNM results, up: the system is acting as an engine, down: the system is acting as a heat pump
There are more parameters that potentially can affect the efficiency of the system. One of these parameters is the length of the heat exchangers. The reason is the length of the heat exchangers directly affects viscous losses. Therefore, it would be interesting to study that effect as well since our goal is find more optimum systems. The simplified numerical model has provided some reliable results and can be used to study the length of the heat exchangers.

For this study, we chose a bigger $x_n$ because bigger $x_n$ leads to greater acoustic velocities which means greater and more obvious viscous effects. Other parameters are: $x_n = 22\,\text{mm}$, $DR = 35\%$, $L_{CHX} = 1.3\,\text{mm}$, $T_C = 300\,\text{K}$, $T_H = 340\,\text{K}$, $x_{hx} = 2.3\,\text{mm}$, $f = 320\,\text{Hz}$, and $y_0 = 0.1\,\text{mm}$. The system is a heat pump. Figure 3.17 shows the second law efficiency of this system as a function of $L_{HHX}$ and as predicted, there is a maximum efficiency which occurs at $L_{HHX} = 0.7\,\text{mm}$.

![Figure 3.17 Variation of second law efficiency with $L_{HHX}$ based on CFD results and SNM results](image)

We use this number to study the performance of the system as a function of $L_{CHX}$. Figure 3.18 shows the variation in second law efficiency with $L_{CHX}$. For Fig. 3.18 $L_{HHX} = 0.7\,\text{mm}$ and $L_{CHX}$ is the variable. The rest of the parameters are the same as those in Fig. 3.17.
Figure 3.18 Variation of second law efficiency with $L_{CHX}$ based on CFD results and SNM results

Again, there is an optimum $L_{CHX}$ for which the efficiency is maximum. Normally, viscous effects decrease with smaller heat exchanger plates. On the other hand, heat transfer rates are directly dependent on the length of the heat exchangers. Therefore, there must be an optimum length for each heat exchanger that results in maximum heat transfer rates with less viscous effects. The maximum efficiency occurs at the optimum length.

In conclusion, the simplified model can be used to predict the optimum geometrical parameters and temperature differences in a no stack thermoacoustic heat pump or engine for a given gas and frequency. Its results have been in agreement with more accurate CFD simulations. This model predicts that no stack thermoacoustic systems can perform with a second law efficiency of 60% as engines or heat pumps.
CHAPTER 4

TRANSVERSE-PIN ARRAY STACK SYSTEMS
Transverse-Pin Array Stack Systems

The no-stack system was introduced as an alternative to the stack system with potentially lower viscous effects. In chapter 1 we explained how a transverse pin stack can be advantageous in terms of reducing the conduction loss thereby increasing the average heat transfer rate from one end to the other. Also, it has the potential to provide more contact surface since they are oriented perpendicular to the acoustic motion. In this chapter, we develop a simplified numerical approach based on Matveev’s model (2010) for a thermoacoustic system with such a stack. In addition we account for temperature fluctuations in the pins by introducing finite heat capacity for the pins into our equations.

Numerical Model

To describe the theory for this setup, we take the same approach as that in chapter 2. We assume that pressure, velocity, and displacement are changing sinusoidally with time and then we find the temperature of the parcel using the first law of thermodynamics for a closed system. The parcel in question is shown in Fig. 4.1.

Imagine that we have a resonator with transverse-pin stack for which the distance from the closed end to the middle of the stack is \( x_n \) and the length of the stack is \( L_{st} \). This stack is sandwiched between two hot and cold heat exchangers and the hot heat exchanger is placed near to the closed end as shown in Fig. 4.2. \( T_H \) and \( T_C \) represent the temperature of the hot and cold heat exchangers respectively. The approach in this chapter is more simplified than that in chapter 2. We only look at one parcel and then we extrapolate the results from our analysis to other parcels. The computational effort will dramatically be less than the model in chapter 2 and 3. The most reasonable parcel to pick is the middle parcel (Fig. 4.2).
Figure 4.1 A small portion of the transverse-pin stack, the gas parcel encloses an array of cylinders in the $y$ direction. (1) is the widest section and (2) represents the narrowest section of the stack.

Therefore, the distance from the closed end to this parcel is $x_n$. Based on the linear acoustic theory from Swift (2002) for any parcel, if $x$ represents the distance from the closed end to the parcel (the same coordinate system as chapter 2) the oscillating complex pressure can be expressed as:

$$ p_1(x) = P_a \cos kx $$ \hspace{1cm} (4.1)
where \( k = \omega / a \) is the wave number and \( P_a \) is the pressure amplitude at the closed end. \( a \) is the speed of sound in the working gas. The complex acoustic velocity can be calculated from the following equation:

\[
\begin{align*}
    u_1(x) &= \frac{-1}{i \omega \rho} \frac{dp_1}{dx} = \frac{P_a k}{i \omega \rho} \sin kx = \frac{P_a}{i \rho a} \sin kx
\end{align*}
\]

(4.2)

The real oscillating properties can be determined by these equations:

\[
\begin{align*}
    p'(t) &= Re(p_1(x)e^{i \omega t}) \\
    u'(t) &= Re(u_1(x)e^{i \omega t})
\end{align*}
\]

(4.3) 
(4.4)

Also:

\[
e^{i \omega t} = \cos \omega t + i \sin \omega t
\]

(4.5)

Therefore, the real oscillating properties will be:

\[
\begin{align*}
    p'(t) &= P_a \cos kx \cos \omega t \\
    u'(t) &= \frac{P_a}{\rho a} \sin kx \sin \omega t
\end{align*}
\]

(4.6) 
(4.7)

For the middle parcel pressure fluctuation and velocity can be specifically written as:

\[
\begin{align*}
    p'(t) &= P_a \cos kx_n \cos \omega t = P_A \cos \omega t \\
    u'(t) &= \frac{P_a}{\rho a} \sin kx_n \sin \omega t = U_A \sin \omega t
\end{align*}
\]

(4.8) 
(4.9)

where \( P_A \) and \( U_A \) are respectively pressure and velocity amplitudes for the middle parcel:

\[
\begin{align*}
    P_A &= P_a \cos kx_n \\
    U_A &= \frac{P_a}{\rho a} \sin kx_n
\end{align*}
\]

(4.10) 
(4.11)

In order to find the displacement \( x \), we take the integral of velocity with respect to time:

\[
\begin{align*}
    x(t) - x_0 &= \int_0^t u'(t) \, dt = \int_0^t U_A \sin \omega t \, dt = - \frac{U_A}{\omega} (\cos \omega t - 1) = -X_A (\cos \omega t + 1)
\end{align*}
\]

(4.12)
where \( X_A = \frac{U_A}{\omega} \) is the displacement amplitude. To find the correct initial position of the parcel, we look at the velocity function: At \( t = 0 \) the velocity is zero and at \( t = 0^+ \), velocity is positive which means the parcel is moving to the right. This means that the correct initial position of the parcel consistent with our equations is the far left. The middle parcel is oscillating about \( x_n \) and the far left position can be determined by subtracting the displacement amplitude from \( x_n \):

\[
x_0 = x_n - X_A
\]

(4.13)

Therefore,

\[
x(t) = x_n - X_A \cos \omega t
\]

(4.14)

Similar to pressure, the displacement consists of two parts: the mean position \( x_n \) and the oscillating part \( x'(t) = -X_A \cos \omega t \). Finally, the pressure, velocity, and displacement, for the middle parcel can be written as:

\[
p(t) = p_m + p'(t) = p_m + P_A \cos \omega t
\]

(4.15)

\[
u(t) = u'(t) = U_A \sin \omega t
\]

(4.16)

\[
x(t) = x_n + x'(t) = x_n - X_A \cos \omega t
\]

(4.17)

The relationships between the amplitudes are:

\[
U_A = X_A \omega
\]

(4.18)

\[
P_A = \rho a Z U_A
\]

(4.19)

\[
\bar{Z} = \frac{\cos kx_n}{\sin kx_n}
\]

(4.20)

Once \( X_A \) and frequency are chosen, parcel’s position, velocity, and pressure are readily known. \( \bar{Z} \) is the absolute value of the non-dimensional specific acoustic impedance which is directly a function of the position of the parcel under discussion in the resonator.
The goal of the simplified model just like chapter 2 is to find the temperature of the parcel at every single time step. The temperature of the middle parcel can be expressed by a combination of a mean value $T_m$ and an oscillating part $T'(t)$:

$$ T(t) = T_m + T'(t) \tag{4.21} $$

Since the parcel in question is right at the middle of the stack between the heat exchangers, $T_m$ will be:

$$ T_m = \frac{T_C + T_H}{2} \tag{4.22} $$

We consider the gas parcel as a closed system and we apply the first law of thermodynamics between $t = t_0$ and $t = t_0 + dt$ where $t_0$ can be any time. If $dQ$ is the heat transfer from the pins in contact with the parcel, the result of this analysis is an equation similar to Eq. 2.17 from chapter 2:

$$ dT = \frac{\gamma - 1}{\gamma} \frac{T}{p} dp + \frac{dQ}{m_p c_p} \tag{4.23} $$

where $m_p$ and $C_p$ are respectively the mass of and isobaric specific heat of the gas parcel. The mean pressure $p_m$ and mean temperature $T_m$ are constant numbers. Therefore,

$$ dp = d(p_m + p') = dp' = -P_A \omega \sin \omega t \tag{4.24} $$

$$ dT = d(T_m + T') = dT' \tag{4.25} $$

Because $T_m \gg T'$ and $p_m \gg p'$ we simplify $T'/p$ as:

$$ \frac{T}{p} = \frac{T_m + T'}{p_m + p'} \approx \frac{T_m}{p_m} \tag{4.26} $$

An expression is required for the heat transfer from the cylinders to the parcel. For steady flow over a bank of cylinders, several expressions for Nusselt number as a function of Reynolds number and Prandtl number are available. Oscillating flow, by nature, is unsteady. However, Swift (1996) and Mozurkewich (2001) showed that if the acoustic displacement is high enough
so that the heat transfer rate exceeds natural convection and can be assumed forced convection. If a thermal penetration length is greater than a characteristic spacing between pins, steady equations can be used for the flow in question. Once the Nusselt number is determined for this flow, the convective heat transfer coefficient and the heat transfer can be calculated as follows:

$$dQ = hA_s dt (T_s - T)$$  \hspace{1cm} (4.27)

where $A_s$ and $T_s$ are respectively the surface area and the temperature of all the cylinders in contact with the parcel. $A_s$ is:

$$A_s = \pi DBN_c$$  \hspace{1cm} (4.28)

where $B$ is the depth of the resonator, $D$ is the diameter of the pin, and $N_c$ is the number of pins that the middle parcel encloses and is calculated as follows:

$$N_c = \frac{H}{S_T}$$  \hspace{1cm} (4.29)

where $H$ is the height of the stack shown in Fig. 4.2 and $S_T$ is the vertical distance between the centers of two adjacent cylinders in $y$ direction shown in Fig. 4.1. $h$ is the convective heat transfer coefficient which will be calculated from the following equation:

$$h = \frac{Nu_D K}{D}$$  \hspace{1cm} (4.30)

where $Nu_D$ is the Nusselt number and $K$ is the conductivity of the gas. To express the changes in heat transfer coefficient for the flow around a bank of cylinders, Zukaukas (1987) provided different empirical correlations for Nusselt number applicable to different ranges of Reynolds number. The following equation from Zukaukas (1987) is the most suitable for this chapter and is valid for $1 < Re_n < 100$:

$$Nu_D = 0.9 Pr^{0.36} Re_n^{0.4}$$  \hspace{1cm} (4.31)

where $Re_n$ is the Reynolds number based on the velocity in the narrowest section of the stack and $Pr$ is the Prandtl number:
\[ Re_n = \frac{\rho u_n D}{\mu} \]  \hspace{1cm} (4.32)

\[ Pr = \frac{\mu C_p}{K} \]  \hspace{1cm} (4.33)

\( \mu \) is the viscosity of the gas and \( u_n \) can be readily derived by writing the continuity equation at the narrowest section and the widest section (Fig. 4.1):

\[ u_n = u' \frac{S_r}{S_r - D} \]  \hspace{1cm} (4.34)

Next step is to find an equation to express the temperature of the cylinders in contact with the parcel. The major difference between this chapter and Matveev’s paper (2010) is that we also account for finite heat capacity of pins, and therefore, temperature fluctuation inside them. This temperature oscillation inside the cylinders does exist in reality because of constant interaction between the oscillatory flow and the solid material. The temperature of the cylinder can be expressed as a combination of a mean temperature and an oscillatory temperature:

\[ T_s(t) = T_{m,s} + T'_s(t) \]  \hspace{1cm} (4.35)

The mean temperature of the cylinders is determined by its position in the stack and the temperature gradient between the heat exchangers. Therefore, from the middle parcel’s point of view:

\[ T_{m,s} = T_m + \frac{dT_m}{dx} x' \]  \hspace{1cm} (4.36)

This equation suggests a linear temperature distribution (as a function of \( x' \)) for only the pins that will come into contact with the parcel during one acoustic cycle. The temperature gradient \( \frac{dT_m}{dx} \) is approximated as follows:

\[ \frac{dT_m}{dx} = \frac{T_C - T_H}{L_{st}} \]  \hspace{1cm} (4.37)
$x'$ is known from Eq. 4.17. Therefore, $T_{m,s}$ is known for all time steps. There are three unknowns at every time step: $T'(t)$, $dQ(t)$, and $T'_s(t)$. Equations 4.23 and 4.27 have been already derived but in order to solve the problem, we need another equation which can be found by considering a cylinder as a closed system and applying the first law of thermodynamics. Using the lumped system analysis for a cylinder and assuming the heat is going out of the system and into the parcel:

$$-\frac{dQ}{dt} = m_s C_s \frac{dT_s}{dt} \quad (4.38)$$

where $m_s$ and $C_s$ are respectively the mass and the specific heat of the pins in contact with the parcel over one time step. From this equation:

$$dT_s = -\frac{1}{m_s C_s} dQ \quad (4.39)$$

From a cylinder’s point of view $T_{m,s}$ is constant because the cylinder is fixed in space and does not move. The only variable temperature with time is $T'_s(t)$. Therefore:

$$dT_s = d(T_{m,s} + T'_s) = dT'_s \quad (4.40)$$

Finally, we have three unknowns and three equations to solve at every time step:

$$dT' = \frac{\gamma - 1}{\gamma} \frac{T_m}{p_m} dp' + \frac{1}{m_p C_p} dQ \quad (4.41)$$

$$dQ = hA_s dt(T_s - T) = hA_s dt \left( T'_s + \frac{dT_m}{dx} x' - T' \right) \quad (4.42)$$

$$dT'_s = -\frac{1}{m_s C_s} dQ \quad (4.43)$$

Since we chase the middle parcel we need to solve the equations from that particular parcel’s point of view. Note that in this case, $T'_s$ is the temperature fluctuation of those cylinders that are in contact with the middle parcel at every time step. In other words, the parcel comes in contact with a group of cylinders and sees their temperature fluctuation at that particular time.
step $T'_s(t)$. Then the parcel passes the previous group of cylinders and sees a new group and $T'_s(t + dt)$ becomes the temperature fluctuation of the new group and this process continues until we finish one cycle.

Next step is to discretize Eqs. 4.41, 4.42, and 4.43. At time step $n$:

$$T'^n - T'^{n-1} = \frac{\gamma - 1}{\gamma} \frac{T_m}{p_m} dp'^n + \frac{1}{m_p c_p} dQ^n$$

(4.44)

$$dQ^n = hA_s \Delta t \left( T'^n + \frac{dT_m}{dx} x'^n - T'^n \right)$$

(4.45)

$$T'^n - T'^{n-1} = -\frac{1}{m_s c_s} dQ^n$$

(4.46)

We left $dQ$ as a variable in the equations because later we need the heat transfer rate to calculate the acoustic power using Rayleigh’s criterion (1945). This set of equations can be organized in the matrix form as follows:

$$\begin{bmatrix}
1 & -\frac{1}{m_p c_p} & 0 \\
hA_s \Delta t & 1 & -hA_s \Delta t \\
0 & \frac{1}{m_s c_s} & 1
\end{bmatrix}
\begin{bmatrix}
T'^n \\
dQ^n \\
T'^{n-1}
\end{bmatrix}
= \begin{bmatrix}
T'^{n-1} + \frac{\gamma - 1}{\gamma} \frac{T_m}{p_m} dp'^n \\
hA_s \Delta t \frac{dT_m}{dx} x'^n \\
T'^{n-1}
\end{bmatrix}$$

(4.47)

These equations need to be solved at every time step and the results will be used in the next time step. At the end of every cycle we must make some modifications to first $T'$ and $T'_s$ of the next cycle in order to obtain reasonable results. We know that the time average of $T'(t)$ and $T'_s(t)$ over one cycle must be zero. By applying this condition we provide better starting values for $T'^1$ and $T'^1_s$ at the beginning of the new cycle. At the end of every cycle, we add the following lines to the code:
After running the code for less than 10 cycles, the results are steady and time average parameters can be calculated.

Since we considered only one parcel, we need to multiply the acoustic power calculated for the middle parcel by the number of parcels. The number of parcels is determined as:

\[
N_p = \frac{L_{st}}{S_L}
\]  
(4.50)

where \(S_L\) is the horizontal distance between the centers of two adjacent cylinders in \(x\) direction shown in Fig. 4.1. Culick (1987) derived a formula for generated or consumed acoustic power based on Rayleigh criterion (1945). In the present configuration, it has the following form:

\[
\dot{E}_{TA} = N_p \frac{\gamma - 1}{\gamma_{pm}} \frac{\omega}{2\pi} \int_0^{\omega} dQ \frac{d}{dt} p' dt
\]  
(4.51)

The heat transfer rate for the middle parcel \(\frac{dQ}{dt}\) is simply \(\frac{dQ}{\Delta t}\) at every time step. To calculate the total heat transfer rate from (or to) the hot exchanger, the equation from Swift (2002) is used:

\[
\dot{Q}_t = \rho C_p A_{st} \frac{\omega}{2\pi} \int_0^{\omega} T'u'dt
\]  
(4.52)

where \(A_{st}\) is the cross sectional area of the resonator calculated as follows:

\[
A_{st} = HB
\]  
(4.53)

In the absence of viscous effects, if \(\dot{Q}_t > 0\), heat is pumped from the hot source to the cold source which is an engine and if \(\dot{Q}_t < 0\), heat is being pumped from the cold source to the hot
source meaning that the system is acting as a heat pump. Based on Swift (2002), $\dot{Q}_t$ is an approximation for $\dot{Q}_H$:

$$\dot{Q}_H \approx \dot{Q}_t$$  \hspace{1cm} (4.54)

To account for viscous loss, first we need an equation that calculates the pressure loss for the flow around an inline bank of cylinders. Usually such equations are available for steady flow while we are dealing with oscillating flow and therefore we are more interested in instantaneous pressure loss. According to Swift (2002), as long as $X_A > S_L$, steady pressure loss equations can be applied to acoustic flow. Gaddis and Gnielinski (1985) proposed the following empirical correlation for steady flow around an inline bank of tubes:

$$\Delta P_{Loss} = N_p \frac{f_a}{Re_n} \frac{u_n^2}{2}$$  \hspace{1cm} (4.55)

where $f_a$ is:

$$f_a = \frac{280\pi}{\left[ \left( \frac{S_k}{D} \right)^{0.5} - 0.6 \right]^2 + 0.75} \left( \frac{S_t}{D} \right)^{1.6}$$  \hspace{1cm} (4.56)

Similar to Eq. 4.31 for Nusselt number, Eq. 4.56 is valid for $1 < Re_n < 100$. Finally, the time average power loss is determined by:

$$\dot{W}_{Loss} = A_{st} \left( \frac{\omega}{2\pi} \right) \int_0^{2\pi/\omega} \Delta P_{Loss} |u'| dt$$  \hspace{1cm} (4.57)

The net generated or consumed acoustic power will be:

$$\dot{W}_{net} = \dot{E}_{TA} - \dot{W}_{Loss}$$  \hspace{1cm} (4.58)

If $\dot{W}_{net} > 0$ and $\dot{Q}_H > 0$ the system is an engine and if $\dot{W}_{net} < 0$ and $\dot{Q}_H < 0$ the system acts as a heat pump. Otherwise it is dissipative. The thermal efficiency, coefficient of performance, second law efficiency will be calculated as follows:
\[
\eta_{th} = \frac{\dot{W}_{net}}{Q_H}
\]

For engines:
\[
\eta_H = \frac{\eta_{th}}{\eta_C}
\]

where \(\eta_C\) is Carnot thermal efficiency.

\[
COP = \frac{\dot{Q}_H}{\dot{W}_{net}}
\]

For heat pumps:
\[
\eta_H = \frac{COP}{\text{COP}_C}
\]

where \(\text{COP}_C\) is Carnot coefficient of performance.

One of the major factors that define the performance of a thermoacoustic system is the temperature gradient between the hot and cold heat exchangers. According to Swift (2002), for a non-viscous system, positive \(\dot{E}_{TA}\) will not be produced unless the temperature gradient exceeds the ideal critical temperature (Eq. 3.6). This brings us to the following dimensionless parameter \(\Gamma\) which is the ratio of actual temperature gradient to ideal critical temperature gradient:

\[
\Gamma = \frac{dT_m}{dx} \frac{\nabla T_{cr,id}}{\nabla T_{cr,id}}
\]

Using Eq. 3.6 along with Eqs. 4.1 and 4.2, the ideal critical temperature will be:

\[
\nabla T_{cr,id} = -\frac{a\omega}{c_p}Z
\]

The negative sign is because the temperature is decreasing in the \(x\) direction. For an actual system with viscous effects, in the vicinity of \(\Gamma = 1\) the system is dissipative. Only when \(\Gamma\) is sufficiently larger than 1 the system performs as an engine. Also when \(\Gamma\) is sufficiently smaller than 1 the system is a heat pump. Since \(\Gamma\) is a dimensionless parameter, it is reasonable to use it
as a key parameter along with three other dimensionless parameters $\frac{S_L}{D}$, $\frac{X_A}{D}$, and $\frac{\rho_s c_s}{\rho c_p}$ to study transverse-pin stack systems.

Results

In the calculation example, we consider the following system: the working gas is air with $f = 1000$ Hz, $p_m = 100$ KPa, $T_m = 400$ K, $L_{st} = 7$ mm, $H = 1$ cm, $B = 1$ cm, $D = 0.05$ mm, and $\tilde{Z} = 5$. We studied the effect of $\Gamma$, $\frac{X_A}{D}$, $\frac{\rho_s c_s}{\rho c_p}$, and $\frac{S_L}{D}$ on the performance of the system. Also, we took $S_T = S_L$. To briefly emphasize the importance of these parameters, it should be mentioned that $\Gamma$ determines the function of the system as a heat pump or as an engine and for a certain value of $\Gamma$ the engine or the heat pump will work more efficiently. The normalized acoustic displacement $\frac{X_A}{D}$ acts as the drive ratio in chapter 2. Bigger acoustic displacement can cause the system to generate more acoustic power or pump more heat from one heat exchanger to the other one. The finite heat capacity of the pins which is studied by changing $\frac{\rho_s c_s}{\rho c_p}$ directly affects the temperature change inside the pins and therefore influences the overall performance of the system. Finally, $\frac{S_L}{D}$ is an interesting geometrical parameter to be examined because it determines Reynolds number (and as a result the viscous loss) and Nusselt number.

The key variable function in this analysis is $p^t \frac{dQ}{dt}$ because the thermoacoustic power is calculated by the taking the time average integral of it (Eq. 4.51). Figure 4.3 shows the variation of this variable over one acoustic cycle for a particular case. The value of this parameter changes
for different values of $\Gamma$, $\frac{X_A}{D}$, $\frac{\rho_s C_s}{\rho C_p}$, and $\frac{S_L}{D}$. Therefore, these are the parameters that will be examined below.

One of the main purposes of developing a simplified numerical model in this chapter was to study how the finite heat capacity of the pins affects the performance of the system. The best non-dimensional number to choose is:

$$\Lambda = \frac{\rho_s C_s}{\rho C_p}$$

(4.63)

We selected a range from $\Lambda = 10$ to $\Lambda = 1000$. Figure 4.4 shows the variation of the second law efficiency with $\Gamma$ for different values of $\Lambda$. The second law efficiency figure has two regions.

![Figure 4.3 $p'\frac{dQ}{dt}$ over an acoustic cycle. $\Gamma = 3, \frac{S_L}{D} = 3, \Lambda = 1000$, and $\frac{X_A}{D} = 25$](image-url)
When $\Gamma$ is sufficiently smaller than 1, the setup acts as a heat pump and when $\Gamma$ is sufficiently greater than 1, it is an engine. Therefore, the right curve represents the engine and the left curve is the heat pump. Each curve has a maximum point which denotes the point where the pump or engine is performing efficiently. The reason lies in formulas 4.51 and 4.52 that determine $E_{TA}$ and $Q_t$. The Carnot efficiency for this system is constant since we are dealing with a constant length stack and in our simplified code we calculate $T_H$ and $T_C$ based on $\Gamma$ and $L_{st}$ using the following simple method:

$$\left| \frac{dT_m}{dx} \right| = \Gamma \left| \nabla T_{cr, id} \right|$$

$$T_H = T_m + \frac{L_{st}}{2} \left| \frac{dT_m}{dx} \right|$$

$$T_C = T_m - \frac{L_{st}}{2} \left| \frac{dT_m}{dx} \right|$$

(4.64)

What changes the second law efficiency is the rate at which $E_{TA}$ and $Q_t$ increase with $\Gamma$ which are shown in Figure 4.5.
These two parameters vary at different rates and therefore the ratios of \( \frac{\dot{E}_{TA}}{\dot{Q}_t} \) for engines and \( \frac{\dot{Q}_t}{\dot{E}_{TA}} \) for heat pumps become maximum for a particular \( \Gamma \). Figures 4.4 and 4.5 also show that for higher heat capacities of the pins the configuration demonstrate better efficiency and is capable of producing more thermoacoustic power and pumping more heat from one heat exchanger to the other. This numerical model predicts that when \( \rho_s C_s \gg \rho C_p \), the maximum second law efficiency for transverse-pin stack system with \( \frac{S_L}{D} = 5 \) and \( \frac{X_A}{D} = 25 \) is around \( \eta_{II} = 50\% \) for a heat pump and \( \eta_{II} = 60\% \) for an engine. This can be explained by the fact that for lower heat capacities of the pins, the interaction between the parcel and the pins causes the temperature of the solid to become closer to that of the parcel and therefore, the amount of the heat transferring from the pins to the parcel decreases according to Eq. 4.42 because \( T_s \) will be closer to \( T \) (Fig. 4.6).
Figure 4.6 Temperature difference between the middle parcel and the pins. For higher heat capacities of the pins this difference is greater which results in greater heat transfer rate between the parcel and the pins. In this figure \( \frac{S_L}{D} = 5 \) and \( \Gamma = 3 \).

This in turn diminishes the amount of acoustic power and total heat transfer rate \( \dot{Q}_t \) thereby decreasing the second law efficiency for the entire range of \( \Gamma \). Figures 4.7 and 4.8 show the variation of efficiency, total heat transfer rate, and thermoacoustic power with increasing \( \Lambda \). It is apparent from these figures that for our current working fluid (air) when \( \Lambda > 500 \), the results do not change noticeably. Thus, greater heat capacity of the solid generally improves the system efficiency and performance.

Figure 4.7 for higher heat capacities of the pins the efficiency reaches its maximum and becomes independent of \( \Lambda \). \( \frac{S_L}{D} = 5, \frac{X_A}{D} = 25 \).
In this study, we also looked at how changing $\frac{S_L}{D}$ can influence efficiency. Increasing $\frac{S_L}{D}$ affects the thermoacoustic power and total heat transfer in more complicated way. On one hand, greater $\frac{S_L}{D}$ leads to smaller friction coefficient as shown in Eq. 4.56 which in turn lowers the viscous loss (Fig. 4.9); on the other hand, $\dot{Q}_t$ and $E_{TA}$ reach a maximum then decrease as $\frac{S_L}{D}$ increases (Fig. 4.10).

Figure 4.8 for higher heat capacities of the pins thermoacoustic power and total heat transfer rate reach their maximum and become independent of $\Lambda$. $\frac{S_L}{D} = 5$, $\frac{X_A}{D} = 25$. 
Despite the decline in $\dot{Q}_t$ and $\dot{E}_{TA}$ after a certain $\frac{S_L}{D}$ the model shows that increasing $\frac{S_L}{D}$ will generally increase the second law efficiency of the system in both heat pump and engine regions (Fig. 4.11).
However, in this model $\frac{S_L}{D}$ has an upper limit and increasing it will lead to unrealistically high second law efficiencies. Its upper limit is determined by the length of the stack $L_{st}$ and the parameter $\tilde{Z}$ which determines the distance from the closed end to the middle parcel. Increasing $\frac{S_L}{D}$ automatically increases the size of the parcel (Fig. 4.1) and that lowers the accuracy of calculations.

Previously, we indicated that lower $\Lambda$ definitely causes smaller thermoacoustic power and smaller total heat transfer rate and as a result smaller efficiency in general regardless how other parameters change. $\frac{S_L}{D}$ was another parameter that was examined along with $\Gamma$. In chapter 2 we introduced the drive ratio as $DR = \frac{P_A}{P_m}$. Here, we also use a geometrical dimensionless number $\frac{X_A}{D}$ which acts as the drive ratio. From equations 4.18 and 4.19, it is obvious that increasing $\frac{X_A}{D}$ for a constant $D$ increases $P_A$ and therefore the drive ratio. Since $\frac{X_A}{D}$ is a geometrical parameter,
it is interesting to study its effect for fixed $\frac{S_L}{D}$, which is another geometrical parameter. To obtain a feasible range for $\frac{X_A}{D}$ we need to bear in mind that it can be written as:

$$\frac{X_A}{D} = \frac{X_A}{S_L} \times \frac{S_L}{D}$$  \hspace{1cm} (4.65)

In order for the parcel to be able to be in contact with at least two adjacent columns of pins over one acoustic cycle, the acoustic displacement must exceed pin spacing, $\frac{X_A}{S_L} > 1$. Matveev (2010) examined this system for $5 < \frac{X_A}{S_L} < 15$. Also, in this work we use $3 < \frac{S_L}{D} < 5$. Therefore, the following range for normalized acoustic displacement is studied here,

$$15 < \frac{X_A}{D} < 75$$  \hspace{1cm} (4.66)

Figure 4.12 shows that except for $\frac{S_L}{D} = 3$ the efficiency of the system does not change dramatically, although increasing the displacement amplitude directly increases the total heat transfer rate and thermoacoustic power (Fig. 4.13).

![Figure 4.12 Variation of efficiency with $\frac{X_A}{D}$ for different values of $\frac{S_L}{D}$. In this figure $\Gamma = 3$ and $\Lambda = 10^3$](image)
Figure 4.13 Variation of thermoacoustic power and total heat transfer rate with $\frac{X_A}{D}$ for different values of $\frac{S_L}{D}$. In this figure $\Gamma = 3$ and $\Lambda = 10^3$

This can be explained with the help of Eq. 4.58 for viscous losses. Figure 4.14 shows the variation of viscous loss with $\frac{X_A}{D}$. It indicates that for $\frac{S_L}{D} = 3$ viscous loss increases at a more significant rate than for $\frac{S_L}{D} = 4$ or $\frac{S_L}{D} = 5$. The decline in the efficiency shown in Fig. 4.12 when $\frac{S_L}{D} = 3$ is caused by greater viscous loss for that particular geometry.

Figure 4.14 The rise in viscous loss by increasing $\frac{X_A}{D}$ for different values of $\frac{S_L}{D}$
Finally, we look at the influence of $\frac{X_A}{D}$ on the general efficiency curve similar to figures 4.4 and 4.11. Figure 4.15 shows how the second law efficiency marginally changes when increasing from $\frac{X_A}{D} = 15$ to $\frac{X_A}{D} = 75$. It can be concluded that increasing $\frac{X_A}{D}$ mostly increases the amount of total heat transfer rate and thermoacoustic power while maintaining almost the same efficiency. However, the direction of this marginal change in the efficiency depends on $\frac{S_L}{D}$. The same analysis was done for $\frac{S_L}{D} = 3$, $\frac{S_L}{D} = 4$, and $\frac{S_L}{D} = 5$. For $\frac{S_L}{D} = 3$ (Fig. 4.15) and $\frac{S_L}{D} = 4$ (Fig. 4.16) increasing the acoustic displacement decreases the efficiency slightly while for $\frac{S_L}{D} = 5$ (Fig. 4.17) slightly greater efficiency is achieved by increasing the acoustic displacement.

![Figure 4.15 Variation of second law efficiency for 15 < $\frac{X_A}{D}$ < 75 In this figure $\frac{S_L}{D} = 3$ and $\Delta = 10^3$](image)
Figure 4.16 Variation of second law efficiency for $15 < \frac{X_A}{D} < 75$. In this figure $\frac{S_L}{D} = 4$ and $\Lambda = 10^3$.

Figure 4.17 Variation of second law efficiency for $15 < \frac{X_A}{D} < 75$. In this figure $\frac{S_L}{D} = 5$ and $\Lambda = 10^3$.

The present model can be used to optimize the system for a particular working gas. For instance, based on the previous results for air a system with $p_m = 100$ KPa, $T_m = 400$ K, $L_{st} = 7$ mm, $H = 1$ cm, $B = 1$ cm, $D = 0.05$ mm, $Z = 5$, $\frac{X_A}{D} = 25$, $\frac{S_L}{D} = 5$, and $\Lambda > 500$ can perform as an efficient engine when $\Gamma = 1.52$ or an efficient heat pump when $\Gamma = 0.77$. 
Conclusion

This thesis is dedicated to numerical study of some alternatives to conventional stack-based thermoacoustic systems. We began with numerical modeling of no-stack configurations. As opposed to conventional thermoacoustic systems, no-stack setups do not include porous material or an array of solid surfaces as a medium for storing and transferring heat. In such systems, heat is transported by gas parcels directly between heat exchangers. A simplified Lagrangian model has been applied to analyze no-stack standing-wave thermoacoustic systems with the aim of estimating beneficial geometrical parameters and acoustic conditions. It was also demonstrated that computational fluid dynamics (CFD) is capable of accurately modeling low-amplitude thermoacoustics. This provided a motivation to employ CFD as a tool to model nonlinear thermoacoustic phenomena for which no exact solutions are available. CFD calculations were carried out to model no-stack systems in favorable conditions more accurately. A qualitative agreement between results of these two approaches was found, but there were also some quantitative discrepancies. These differences are attributed to inadequate modeling of friction loss, not accounting for transverse temperature variation, neglecting viscous heating, and heat conduction between gas parcels, in the simplified model.

Using the CFD code FLUENT, efficient setups were found with the second law efficiency of 55% for a heat pump and a prime mover. The dependence of the system performance is reported for variable system parameters. We showed how a system can evolve from a heat pump to an engine by changing the distance from the plates to the closed end of the resonator or by adjusting a temperature difference between the heat exchangers. Heat exchangers with finite-thickness plates were additionally investigated in high-performance configurations. For porosities below 25%, the thickness effect appeared to be insignificant.
In another attempt, a modified approach towards numerical modeling of one type of intermittent stacks was undertaken. The transverse-pin array stack systems have the potential to increase the efficiency of thermoacoustic power conversion compared to systems with usual stacks. One reason is that the transverse-pin setup reduces conduction losses via solid material. In addition, pin-stack configurations demonstrated better power density in comparison with circular pores in conventional stack-based setups. In this study, the transverse-pin system was modeled numerically with a simplified method. For computational efficiency, only one gas parcel was traced. The thermoacoustic power conversion was calculated using the Rayleigh criterion. The results were extrapolated to other parcels between the heat exchangers. Viscous loss was accounted for using the steady state friction factor for a bank of cylinders. Also, the finite heat capacity of the pins was taken into account. When pins’ heat capacity is small, significant temperature fluctuations in the solid material appear negatively affecting the overall efficiency of the system. A number of parameters were examined and the numerical results showed that this configuration can work with a second law efficiency of 60% (as a heat pump or a prime mover) and the amount of acoustic power can be increased by using higher drive ratios. The code developed based on this theory can be used for parametric design of transverse-pin-stack thermoacoustic systems.

Computationally efficient models were developed in this work for modeling novel thermoacoustic setups. It was found that both transverse-pin array stack and no-stack systems have the capability of having greater power density compared to conventional stack-based systems. These numerical models can be used to find high-performance regimes and configurations, which more accurate optimization can be accomplished with CFD tools. The present research can be extended in the future by formulating more accurate numerical models.
with additional components of thermoacoustic engines. Experiments with studied here configurations are needed for additional model validation and practical implementation of novel thermoacoustic devices.
References


