QUANTUM HYDRODYNAMICS IN ONE- AND TWO-COMPONENT
BOSE-EINSTEIN CONDENSATES

By

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Abstract

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Several prototypical experiments concerning quantum hydrodynamics are realized in this thesis using one- and two-component Bose-Einstein condensates (BECs). The experiments are conducted with an experimental apparatus built at WSU that is capable of reliably producing $^{87}$Rb BECs and $^{40}$K degenerate Fermi gases (DFGs). The apparatus, which has undergone many modifications and upgrades since it was first built, will be described in detail. The upgrades include the addition of fermionic potassium atoms, installation of a fully electromagnetic Ioffe-Pritchard type trap with excellent optical access to the BEC, and the addition of an optical dipole trap (and optical lattices).

In the first set of experiments, I describe studies in which the dynamics of merging and splitting single component BECs lead to the observation of dispersive shock waves and soliton formation. In splitting a BEC, the transition from sound wave excitations to dispersive shock formation is examined. Motivated by our single component BEC experiments, we go on to study superfluid-superfluid counterflow using BECs containing two different hyperfine states. Surprisingly rich dynamics are observed for counterflow speeds exceeding a critical velocity. Above this critical velocity, a counterflow-induced modulational instability sets in and drives excitations in the form of dark-bright solitons and novel oscillating dark-dark solitons, which have previously been theoretically described (e.g. in the context of nonlinear optics), but never before been observed in the laboratory.
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For my parents.
Chapter 1

Introduction

We live in a classical world in which most people are accustomed to picturing atoms as being particle-like in nature. However, as we explore deep into the microscopic world, quantum mechanics takes over and the laws of classical physics melt away. In the quantum world, the wave nature of atoms become decidedly important and the distinction of whether an atom is a boson with integer spin or a fermion with half-integer spin cannot be swept aside. Nothing prevents a group of bosonic particles from occupying the same single particle energy state all at once whereas fermionic particles must obey the Pauli exclusion principle and no two identical fermions can occupy the same energy level. In the 1920s, Satyendra Nath Bose and Albert Einstein developed a theory applicable to a gas of ideal bosons in which it was predicted that, at low enough temperatures, a finite fraction of all particles in the system would fall into the lowest available energy level, forming a state of matter now known as the Bose-Einstein condensate (BEC)\(^\text{1}\).

A useful parameter in characterizing the condensation of bosons is the phase space density, \(n\lambda_{dB}^3\), in which \(n\) is the number density and \(\lambda_{dB}\) is the thermal deBroglie wavelength given by

\[
\lambda_{dB} = \sqrt{\frac{2\pi\hbar^2}{mk_B T}}
\]  

\(\text{(1.1)}\)

\(^1\)For a more complete historical account of BEC see, for example, the 2001 Nobel Prize Lectures.
with $m$ the particle mass, $k_B$ the Boltzmann constant, $\hbar$ Planck's constant divided by $2\pi$ and $T$ the temperature. For a uniform ideal Bose gas in which the particles are non-interacting, the BEC phase transition occurs at a phase space density of $\sim 2.6$.\footnote{This order of magnitude is understandable from an intuitive picture where atomic wave packets must overlap in order to form a BEC.} In practice, bosonic gases employed in the study of BECs always involve a finite number of atoms and are almost always confined in harmonic traps with non-uniform density distributions. Interatomic interactions also come into play in the real world and can lead to significant differences in the behavior of a BEC from the ideal gas instance. In any case, the transition to a BEC is observed as a sudden and pronounced peak in the momentum distribution of the particles.

The first experimental realization of a dilute gas BEC was accomplished in 1995 by the groups of Eric Cornell and Carl Wieman at JILA, the University of Colorado using $^{87}$Rb atoms, and by Wolfgang Ketterle’s group at MIT using $^{23}$Na atoms. Condensates of $^{87}$Rb are particularly popular among experimentalists because of its favorable scattering properties which make it “easy” to condense, and because the D1 and D2 lines of this isotope (at 795 nm and 780 nm respectively) happen to coincide with the lasing transitions of widely available (and cheap) diode lasers found in DVD and CD players. For the same reasons, the fermionic isotope of potassium, namely $^{40}$K, is used in many laboratories to study degenerate Fermi gases (DFGs).

Since the first creation of dilute gas BECs, hydrodynamic experiments have played a crucial role to elucidate the nature of this novel quantum state. The very first experiments focused on the collective excitations [1, 2] of a condensate in harmonic traps. Soon after, interference between two expanding condensates was observed [3], providing experimental confirmation of the wave-nature of dilute gas BECs. Quantized vortices in rotating BECs [4, 5, 6] supplied proof of superfluidity in the gas. Recent experiments involving hydrodynamic defect structures (topological defects) in BECs include work in which dark [7, 8] and dark-bright [9] solitons were generated using phase-imprinting methods. Bright soliton trains have also been observed in condensates with attractive interactions [10]. In a two-dimensional
setting, vortices which form during the BEC phase transition have been studied [11, 12] and vortex-anti-vortex pairs created by forcing a BEC to flow past a barrier [13] have been observed. Vortices and specifically, vortex tangles, are one of the telltale signs of classical turbulence. In BECs, vortex tangles have been observed in [14] and provide an exceptional opportunity to study quantum turbulence [15], which is theoretically more tangible than classical turbulence.

The hydrodynamics of ultracold Fermi gases have also been of theoretical [16] as well as experimental interest [17]. As with bosons, observation of vortices in a strongly interacting cloud of fermions [18] confirms the superfluid nature of ultracold Fermi gases.

Nonlinear structures such as vortices and solitons have been studied with enthusiasm as they are familiar excitations found in many different physical systems ranging from plasma physics, nonlinear optics, to water waves and condensed matter systems. In this short account, many other fascinating hydrodynamic phenomena must remain unmentioned, including vortex rings, large vortex lattices, Faraday waves, etc.\(^3\)

In the context of BECs and DFGs, a major motivation for the study of vortices, solitons, and other quantum hydrodynamic phenomena is the quest to understand the peculiar properties of superfluidity. Superfluidity, the ability of a fluid to flow without friction below a critical velocity\(v_c\), is doubtlessly one of the most fascinating hydrodynamic properties of a gas of condensed atoms. Much theoretical and experimental effort has been dedicated to studying this phenomenon since its discovery in superfluid \(^4\)He by Kapitsa [21], Allen and Misener [22] in 1938. In the early 1940s, Tisza and Landau formulated the two-fluid model [23] in which superfluid \(^4\)He is thought of as being a mixture of two fluids containing a superfluid component and a normal component. Prior to the realization of BECs, early studies of quantum hydrodynamics concentrated exclusively on superfluid helium systems. As a prominent example, quantum turbulence was studied using the counterflow between the normal and superfluid components. However, superfluid helium is notoriously difficult to work with because of strong interatomic interactions and, consequently, the small physical properties.

\(^3\)For an in-depth review of hydrodynamic phenomena, see [19]. For experiments demonstrating Faraday waves in BECs, see [20].
sizes of features such as vortex cores. In addition, little research has focused on the dynamics resulting from counterflow between two superfluids.

There are several advantages to studying quantum hydrodynamics using alkali gas BECs. First, these quantum systems are macroscopic and can easily be observed with a simple microscope imaging system as compared to superfluid helium. The lower density of dilute gas BECs leading to a longer healing length allows for direct observation of hydrodynamic defect features such as solitons and vortices. Second, their response to electric and magnetic fields provide experimentalists with the ability to design flexible trapping potentials\(^4\) and many different ways to induce excitations in the system. Third, the remarkable control of system parameters such as the number of condensed atoms, internal degrees of freedom, interatomic interaction strengths, etc. provide a very clean and simple system to work with. These advantages combine to make dilute gas BECs an ideal playground with which quantum hydrodynamics can be studied.

### 1.1 Thesis Organization

This thesis is organized as follows. Chapter 2 describes many different aspects of building and maintaining our experimental apparatus. In Chapter 3, we present a discussion of how raw data collected by our BEC apparatus is analyzed to extract useful information about our system. The main results of this thesis are presented in Chapters 4 and 5. Numerous theoretical studies involving supercritical flow (see, for example, [25, 26]; and references therein) have demonstrated the shedding of defect structures such as solitons and vortices following the formation of large density modulations analogous to classical shock waves. We study the formation of these quantum shock waves, also called dispersive shock waves (DSWs), in Chapter 4 by merging and splitting two BECs in the nonadiabatic, high-density regime. The shedding of dark solitons is observed following the formation of DSWs. We

\(^4\)Trapping potentials can be one-, two- or three-dimensional. Periodic potentials (optical lattices) can also be realized using retro-reflected, or otherwise interfering, laser beams. In addition, arbitrary “painted” potentials have also recently been realized [24].
first introduce the experimental procedure followed by results and discussions. An outlook on the interesting dynamics involving DFGs is presented in the last section of this chapter.

The experiment presented in Chapter 5 extends the study of Chapter 4 by investigating the dynamics of superfluid-superfluid counterflow using BECs composed of a mixture of atoms in two different hyperfine states. The two components are made to flow through each other in a narrow channel and strikingly rich dynamics are revealed. A counterflow induced modulational instability is determined to be the underlying generator of vector solitons. Different types of solitons, including trains of dark-bright solitons and a novel oscillating dark-dark soliton, are observed when different boundary conditions are satisfied. We also present in this chapter studies exploring the in-trap oscillations of a single dark-bright soliton, initial investigations of the interactions between two dark-bright solitons as well as interactions between dark-bright solitons and repulsive barriers.

Included in the appendices of this thesis are descriptions of technical details related to a second ultra-cold atoms machine that is currently being completed in our lab. The new machine will be able to perform experiments using three different elements: $^{40}$K, $^{87}$Rb and $^6$Li. Appendix A.1 includes a description of how we make our own enriched $^{40}$K dispensers. In Appendix B.1 we report on our experiences with building a lithium heat pipe spectroscopy cell and in Appendix B.2, we describe the construction of a lithium oven as a source of atoms in the new machine. In Appendix C.1, circuit diagrams of the electronics used throughout our lab are provided.
Chapter 2

Experimental Apparatus

2.1 Introduction

This chapter describes our ultracold atoms machine with which all experiments in later chapters of this thesis are performed. BEC machines are designed and built to meet specific experimental needs and over the years, our machine has grown tremendously. Starting out as a single species machine capable of producing $^{87}\text{Rb}$ condensates in a hybrid Ioffe-Pritchard (HIP) trap, it has gone through several stages of evolution. Improvements to the machine include the addition of $^{40}\text{K}$ (fermions), upgrading the HIP trap to a fully electromagnetic Ioffe-Pritchard trap and the installation of an optical dipole trap which has allowed for our two-component experiments, a 3D optical lattice, a 2D superlattice, as well as a Raman laser system for spin-orbit coupled BEC experiments (to be described in a future thesis).

The entire machine is housed across three $4' \times 8'$ TMC optical tables, two of which are kept floating to help dampen out vibrations. One table is dedicated entirely to laser systems, with half of it occupied by lasers for rubidium and the other half by lasers for potassium. A second table houses the vacuum chamber along with some optical components for laser cooling and imaging while on the third table we prepare the beams for our optical dipole trap, optical lattices, and the Raman system.
2.2 Vacuum System

A vacuum chamber capable of maintaining ultra-high vacuum (UHV) pressures is essential in the production of a BEC. Contaminant particles within the chamber are not cooled and, if not removed, will heat the sample of atoms to be condensed so that it will be impossible to reach BEC. The main sections of our vacuum system consist of chambers holding atomics sources, a glass magneto-optical trap (MOT) cell, a differential pumping stage, a Pyrex science cell, two ion pumps, and a titanium-sublimation (Ti-sub) pump. Figure 2.1 shows a schematic overview of the vacuum chamber.

Atoms released from alkali dispenser sources (or ampoules) are captured and laser-cooled in the MOT cell. These precooled atoms are transferred from the MOT cell to the science cell for further evaporative cooling down to degeneracy. The design of our vacuum chamber is such that a double MOT setup is not needed, thus freeing us from the need for more optics and lasers. The comparatively large background pressure of Rb and/or K atoms in the MOT chamber (about $10^{-9}$ Torr) originates from the proximity of the alkali sources and supports reasonable capture rates of the MOT. The much lower background pressure in the science cell (about $10^{-12}$ Torr) ensures that evaporative cooling to BEC is possible. The pressure difference between the two chambers is maintained by a differential pumping stage.

Ultra-high vacuum pressures are achieved with standard bakeout procedures of the chamber and its pumps [27]. We follow a procedure in which all parts used in the construction of the chamber, with the exception of glass cells, viewports, pumps, and the all-metal valve are first cleaned in an ultrasonic bath containing a solution of deionized water and Micro-90 for about 20 to 30 minutes, rinsed with deionized water, rinsed with ultrapure spectrophotometric grade acetone, dried with spectrophotometric grade methanol, then prebaked at a temperature of about 350°C for about 4 to 5 days\(^1\). These parts are then wrapped in oil-free UHV foil from All-Foils Inc. and stored in a dust-free environment until they are ready to be assembled. It is important to wear powder-free latex gloves at all times while handling UHV parts to avoid contaminating the surfaces with oil/dirt/powder from the hands.

\(^1\)originally 4hr at 400°C
The assembled vacuum chamber is first wound with tape heaters (from OMEGA) and thermocouples are placed strategically at different locations of the chamber. It is especially important to monitor the rate at which temperature is changing at the delicate glass and/or quartz parts. Increasing (or decreasing) the temperature too fast could lead to leaks/cracks in the windows and glass chambers. The machine is then wrapped in many layers of crumpled aluminum foil for insulation. The tape heaters are powered with variacs which allow for the slow increase of temperature. At the same time, we can adjust for temperature differences at different parts of the chamber. Two bakeouts of the assembled vacuum chamber are performed. The first high temperature bakeout is performed without delicate components such as viewports and glass cells. Starting from room temperature, the entire chamber is brought to a temperature of about 300°C over the course of a week or so at a rate of \( \sim 10^\circ \text{C/hour} \). The temperature is then kept at 300°C for one to two weeks before it is brought back down to room temperature at the same rate. A second bakeout at a lower temperature (200°C for about a week) follows after glass components have been attached. The rate at which we increase and decrease the temperature of the second bakeout is about 5°C/hour. After the second bakeout, if no major leaks have developed and the chamber well-outgassed, an ion pump reading of \(< 1 \times 10^{-7} \text{ A} \) at 3 kV is achieved.
2.2.1 Vacuum Pumps

During the initial machine bakeouts, a turbo pump (Pfeiffer Vacuum TMH 071) backed by an oil-free piston pump (Pfeiffer Vacuum XtraDry 250-2) evacuates the chamber to achieve pressures down to about $10^{-9}$ Torr. The Ti-sub pump is turned on once for degassing during the final bakeout. We run 42 A (at 4.5 V) for a duration of 3 minutes through each of the three filaments at a temperature of about 100°C during the cooldown of the chamber. The ion pumps are turned on periodically during the cooldown to check the progress of the bakeout and, based on the ion pump current, provides an indication of the vacuum quality. We do not turn on the ion pumps fully until after the end of the second bakeout.

Once the machine is operational, the turbo and forepump are valved off with an all-metal UHV valve and are switched off (these pumps can be removed if they are needed elsewhere). Both ion pumps (from Varian Inc. now Agilent Technologies) work continuously to pump particles out of the vacuum chamber to help maintain UHV pressures. A 20 l/s ion pump (VacIon Plus 20 StarCell) works to dispose of rubidium and/or potassium atoms directly before the differential pumping stage. A vapor pressure on the order of $10^{-9}$ Torr is maintained in the MOT cell, close to the atomic sources\(^2\). A 55 l/s ion pump (VacIon Plus 55 StarCell) after the differential pumping stage and close to the science cell pumps “used” rubidium atoms out of the vacuum system to keep the pressure in the science cell on the order of $10^{-12}$ Torr. Both ion pumps are controlled by a MidiVac Ion Pump controller (also from Varian). We continuously monitor the pump current to ensure that, at an operational voltage of 3 kV, the pump current stays below $2 \times 10^{-7}$ A.

A filament type Ti-sub pump and controller from Varian is used to provide additional pumping for getterable gases such as hydrogen and nitrogen. Unlike the ion pumps, the Ti-sub pump is only turned on on an as-needed-basis. If the the ion pump current rises to above the usual 0 to $1 \times 10^{-7}$ A at 3kV, we will activate the Ti-sub pump by running a current of 48 A (5 V) for 1.5 minutes through one of the filaments to clean the system out.

\(^2\)A systematic treatment of the vapor pressure of a two-species MOT (Rb and K) is given in [28]. See also equation (1) in [29].
This is done once after the initial bakeout, and only very rarely after that (about once every couple of years).

### 2.2.2 MOT Chamber

In the original version of our BEC machine, a MOT cell with a circular cross section was used (figure 2.2). We made the decision to switch to a square cross section MOT cell in order to minimize distortions of the cooling beams upon entering the cell. At the time of the machine upgrade, we discovered that the manufacturing process for producing a large square cell was difficult and costly. The most cost effective solution for us at the time was to purchase a square cell made of borosilicate (2.8 mm wall thickness, 51 mm ID) from Friedrich & Dimmock and send the cell to Technical Glass/Precision Glassblowing in Centennial, CO for attaching the necessary flanges. The square cell was made using a process in which the glass is drawn into its final shape. This manufacturing process left visible streaks in the glass across the sides of the cell but luckily they are not centered in the faces and we do not see any noticeable distortions in our cooling beams. In the end, the square MOT cell did not noticeably improve the performance of the cooling beams.

As can be seen on the left of figure 2.3, a glass-to-metal seal attaches the cell to a bellows section for relieving strain on the glass when the flanges are tightened. A second glass-to-metal seal on the opposite end of the cell is directly attached to a 2-3/4" CF flange. The approximate dimensions of our MOT cell is given in this figure. Also visible in this picture are our atomic source dispensers sitting in the end opposite to the bellows (see section 2.4.1).

### 2.2.3 Differential Pumping Stage

The differential pumping stage is a 6" long piece of 0.49" ID stainless steel tubing. Two 2-3/4" CF flanges are attached to either end of the tube. The small diameter of the tube prevents the atoms at higher pressures in the MOT cell from entering the science cell. It also serves to confine the higher pressure in the MOT cell so that we have sufficient MOT capture rates.
Figure 2.2: Original MOT cell with circular cross section made of 1/8” thick glass. This cell has an outer diameter of 2” and is 12.5” long.

Figure 2.3: Picture of upgraded MOT cell showing MOT/transfer coils and dispensers sitting in the tubular section of the cell on the right. Optical pumping coils are also visible.
For a long tube with a circular cross section, the conductance in the molecular flow regime is given by [27]

\[ C = \frac{\pi}{12} \frac{v^3}{l} = \frac{\pi}{12} \sqrt{\frac{8k_B T}{\pi m}} \frac{d^3}{l} \quad (2.1) \]

where \( v \) is the average velocity of the particles, \( m \) is the particle mass, \( k_B \) is Boltzmann’s constant, \( T \) is the temperature, \( d \) the diameter of the tube, and \( l \) the length of the tube. Assuming the tube is at room temperature (293 K) and rubidium atoms with an average molecular weight of \( m \approx 85.5 \) g/mol, the conductance becomes

\[ C = 45.5 \frac{d^3}{l} \text{ (in units of l/s)} \quad (2.2) \]

where \( d \) and \( l \) are in inches. For air molecules, \( m \approx 29 \) g/mol and the conductance becomes \( C = 78d^3/l \). Substituting the dimensions of our differential pumping stage into the above formula gives a conductance of 0.89 l/s for Rb (and 1.5 l/s for air). Considering just the 55 l/s pump after the differential pumping stage, the pumping speed is \( S_{\text{pump}} = C(P_1 - P_2)/P_2 \) where \( P_1 \) is the pressure on the MOT cell side of the tube and \( P_2 \) is the pressure on the science cell side of the tube. Rearranging, the ratio of the two pressures can be expressed as

\[ \frac{P_2}{P_1} = \frac{C/S_{\text{pump}}}{1 + C/S_{\text{pump}}} \approx \begin{cases} 0.02 & \text{for rubidium} \\ 0.03 & \text{for air.} \end{cases} \quad (2.3) \]

### 2.2.4 Science Cell

The science cell of the original machine was made of uncoated 1 mm thick Pyrex [fig. 2.4(a)] manufactured by Starna Cells. A glass-to-metal seal purchased from Larson Electronic Glass was joined to the science cell by Ron Bihler of Technical Glass, Inc. When we upgraded our science trap from the HIP trap to the fully electromagnetic trap (see section 2.6), a new, slightly larger science cell became necessary due to the different trapping axis orientation of the new trap. The upgraded science cell [fig. 2.4(b)] was also purchased from Starna Cells and has a MgF AR coating optimized for 1064 nm on the outside to help reduce unwanted
Figure 2.4: (a) Original Pyrex science cell is uncoated. (b) Upgraded Pyrex science cell is AR coated on the outside only. (c) Aluminum shield for protecting the science cell during vacuum bakeout.

reflections of the optical dipole beam, lattice beams, etc. Both cells are attached to a 2-3/4” CF flange.

During the machine upgrade vacuum bakeout, the AR coating of our science cell was badly damaged by heat. The cell had been directly wrapped with a thin layer of aluminum foil over which heater tape was wound and baked at about 200°C for a week and a half. The damage to the coating was severe enough to warrant a second attempt with a new cell. Though frustrating and time consuming, other issues (involving malfunctioning commercial dispensers) also demanded that we break the vacuum. To protect the AR coating during the second attempt, we designed a cylindrical shield made of aluminum into which the science cell was inserted [figure 2.4(c)]. Heater tape can then be wrapped around this shield and no direct contact is made with the science cell itself. It is important to make sure that the shield is well wrapped and closed with many layers of aluminum foil. This removes the risk of cold air being blown into the shield during the bakeout, which could easily crack the glass.
2.3 Laser Systems

2.3.1 Rubidium Laser System

Our original rubidium laser system consisted of three commercial diode lasers (New Focus TLM 7000 Vortex lasers), one was used for the MOT cooling, another for the repump beams, and the last for imaging the atoms. The MOT laser was operated at 75 mA, giving an output power of \( \sim 28 \) mW. The repump and imaging ECDLs were operated at 70 mA, giving an output power of \( \sim 26 \) mW (all lasers are deliberately operated well below their maximum specified output power of 50 mW in the hopes of extending diode lifetimes). About four years after the machine became operational, we ran into a problem with the Vortex laser used for imaging. The Rb absorption lines had slowly drifted (over a time scale of a few months) out of the laser’s adjustable piezo voltage range which forced us to send the unit back to New Focus for an expensive and time consuming cavity realignment (the New Focus lasers do not have any accessible parts or adjustment possibilities). Over the following year, the same laser drifted off the lines again. Shortly after the laser cavity realignment fiasco, a second
Vortex began drifting out of range. One hypothesis is that the drifts are caused by aging AR coatings on the laser diodes. Not wanting to limit our experiments by the downtime brought about by having to once again send a second laser back for realignment, much effort was spent on designing homebuilt external cavity diode lasers (ECDLs), see fig.2.5, and incorporating them into our setup [30]. The original imaging Vortex was swapped out with the repump Vortex and a homebuilt ECDL is currently used in the repump setup with an operating current of 55 mA, giving about 22 mW of power out. Using a Vortex for imaging is convenient because its large mode hop free tuning range provides easy switching between the F=1 and F=2 lines for probing the atoms. The final design of our homebuilt ECDLs is in the Littrow configuration, uses an Invar flexure mount and is temperature controlled using two peltiers. We use homebuilt temperature and current controllers based on Wavelength Electronics modules to run these lasers (see Appendix C.1). Soon after the replacement of the repump Vortex with a homebuilt laser, we were forced to replace the MOT Vortex as well. The homebuilt MOT ECDL is operated with a current of 75 mA and outputs 30 mW of power. Currently, our Rb laser system uses one Vortex laser (imaging) and two homebuilt lasers (repump and MOT beams). Both homebuilt ECDLs use non-AR coated Mitsubishi ML601J38 diodes which were purchased for about $10 each. These homebuilt lasers turned out to be extremely user friendly. They remain frequency locked throughout the entire day and have not required cavity alignments for months.

A small amount of the output light from each laser is branched off for standard saturated absorption spectroscopy, which we use to resolve the various hyperfine absorption lines of rubidium [31]. Schematic diagrams of the optical layout of the MOT (cooling) and repump laser systems are shown in Figure 2.6 (a) and (b). We boost the power from the MOT ECDL by seeding an Eagleyard tapered amplifier (TA) chip (EYP-TPA-0780-00500-3006-CMT03-000, 500 mW) with 15 mW of power in a single pass configuration to obtain about 350 mW (driving the TA chip at 1 A) in the cooling beam prior to sending the light into a fiber. This TA chip has been operational since 2006 and we have not seen any degradation of its performance yet.
Figure 2.6: Optical layout of Rb laser system for (a) MOT and (b) repump. The saturated absorption spectroscopy is contained within the blue dashed boxes. (c) block diagram of locking electronics.
Figure 2.6 (c) shows a simplified block diagram of the locking electronics. The signal from the saturated absorption spectroscopy along with its error signal can be monitored on an oscilloscope as seen in figure 2.7 while the piezo is scanned across the absorption lines using a ramp generator set to a triangle wave. The derivative of the absorption lines are created using a dither box which weakly modulates the laser current at a frequency of 300 kHz. The dither box then demixes the 300 kHz modulation from the saturated absorption signal. This can be thought of as a single frequency lock-in type setup with the output generating the locking signal as the piezo is scanned across the lines. The design of our dither box can be traced back to JILA designs.

We lock the MOT laser to the transition between the $5S_{1/2}, F = 2$ and the $5P_{3/2}, F' = 2 \rightarrow 5P_{3/2}, F' = 3$ crossover peak [see fig. 2.7 (b) and fig. 2.23(a) for $^{87}$Rb energy level scheme] using light from the -1st order of a 120 MHz AOM. Light from the 0th order of this AOM is sent to the MOT. The cooling light hitting the atoms is slightly red detuned (by about 13 MHz) from the $5S_{1/2}, F = 2 \rightarrow 5P_{3/2}, F' = 3$ cycling transition. The repump laser
is locked directly to the $5S_{1/2}, F = 1 \rightarrow 5P_{3/2}, F'' = 2$ transition to pump atoms which have fallen out of the cycling MOT transition back into the cooling loop. About 8 mW of power (out of the fiber) is supplied for the repump beam.

Light for optical pumping is branched off from the MOT laser just before the beam enters the TA. We use the -1st order beam from a 260 MHz AOM to shift the optical pumping light to the $5S_{1/2}, F = 2 \rightarrow 5P_{3/2}, F' = 2$ transition. About 0.25 mW shines onto the atoms in the MOT during the brief optical pumping phase (see section 2.4.5).

The imaging laser system will be described in section 2.10.

### 2.3.2 Potassium Laser System

The optical layout for the potassium lasers is shown in figure 2.8. Two homemade ECDLs, each housing diodes from Eagelyard\(^3\) and outputting about 8 mW of power, are used to generate light for the MOT, repump, optical pumping and imaging beams.

The energy levels for $^{39}$K and $^{40}$K (D2 lines only) are sketched in figure 2.9(a)\(^4\) where the optical transitions for the locking, MOT, and repump are indicated. We use a 210 MHz AOM in a double pass configuration to shift a portion of the light originating from the repump ECDL up in frequency by $\sim$420 MHz (blue beam path in fig 2.8). This light is then sent to the saturated absorption spectroscopy portion of the setup (using PBS 1) where it is used to lock the repump laser to the $^{39}$K, F=2 absorption peak. The portion of light which has not been shifted by the AOM (red beam path in fig 2.8) sits at a frequency 420 MHz below the $^{39}$K, F=2 transition, which is where the repump transition for $^{40}$K needs to be. The repump light is then combined with light from the MOT/imaging laser (green beam path in fig 2.8) at PBS 2 and a small portion is sent to a fast photodiode for offset locking. The rest of the combined light is amplified using an Eagleyard TA at 765 nm (EYP-TPA-0765-01500-3006-CMT03-0000) in a double-pass configuration [33]. The TA is seeded from the “wrong” side, i.e. light is injected into the tapered end and passed back through the normal input end.

\(^3\)Originally, EYP-RWE-0790-04000-0750-SOT01-0000 diodes were used but these died quickly at the higher output powers we needed. We now use EYP-RWE-0780-02000-1300-SOT02-0000.

\(^4\)A more detailed diagram can be found in [32].
Figure 2.8: Optical layout for potassium laser system. Saturated spectroscopy used for locking the lasers is outlined in dotted box. Double-pass AOM outlined in dashed box.
Figure 2.9: (a) Level scheme for $^{39}$K and $^{40}$K. (b) Absorption lines from an unenriched potassium vapor cell. Due to the low abundance of $^{40}$K, the $^{40}$K absorption lines do no show up in (b).

Very little seeding power is needed in this configuration. No more than 50 mW should be sent towards the input side of the chip and the seeding power should be $\ll 25$ mW. Hence, attenuators were necessary in reducing the seeding power to a total of about 0.6 mW. To monitor the power on the first pass through the TA, a pickoff is used to send some light to a photodiode. It is also important to monitor the first pass power while turning the TA on. Once the TA is up to our operating current of 2.1 A, it is very stable. We keep the power after the first pass through the TA below 14 mW. At high powers, if the retro-reflected beam (second pass) is accidentally blocked, the power through the first pass could be amplified to a point which damages the TA chip. We get about 380 mW of output power in total out of the TA. About 200 mW is output to the MOT/repump and $\sim 1.25$ mW is output for optical pumping (though only 1/3 of this power makes it to the atoms).

We image potassium on the MOT transition. An 80 MHz AOM in the imaging beam path is essentially used as a fast shutter. The frequency shift of this AOM is compensated by changing the offset lock. To offset lock the MOT/imaging laser, the beat frequency between light from the MOT/imaging laser and light from the repump, which needs to be at 1241.7 MHz (minus some red detuning for the MOT), is mixed with the signal from a 1090 MHz VCO. The output, at about 150 MHz, is sent to a frequency discriminator where it is compared with the output from a variable VCO whose frequency is set at 151.7 MHz.

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5 We use a PLL box design which has a digital frequency/phase discriminator. However, we only use the frequency discriminator.
(without MOT detuning). The frequency of the MOT/imaging laser is servoed so that the difference between the mixer output and tunable VCO is zero.

Figure 2.9(b) shows the saturated absorption spectroscopy signal from an unenriched K vapor cell made by Precision Glassblowing in Denver, CO. To increase the vapor pressure, the cell is heated up to 50 °C and signals from $^{39}K$ and $^{41}K$ are clearly visible. The natural abundance of potassium isotopes are 93.2581% for $^{39}K$, 0.0117% for $^{40}K$, and 6.7302% for $^{41}K$ [34]. Thus, the absorption signals from $^{40}K$ are too small to be seen in figure 2.9.

### 2.3.3 Repulsive and Attractive Barriers

Beyond the initial preparation of the ultra-cold gases, similar diode based lasers are used to generate optical fields to spatially probe the degenerate gases. An essential tool used for the hydrodynamic experiments described in Chapter 4 is a single beam dipole barrier far-detuned from the Rb and K D-lines. Initially, an AOM provided this dipole potential with the capability of being able to sweep across a cloud of atoms. Later, piezo stacks attached to a mirror mount replaced the AOM to provide a larger spatial sweeping range. There are two such dipole beams in our lab, one being red-detuned and another being blue-detuned from the Rb and K transitions, but both share the same optical setup. Since only one beam is needed at a time, we switch between the two at the input end of the optical setup as sketched in figure 2.10. The dipole barrier is sent horizontally through the center of the HIP trap, along the radial (tightly confining) y-direction. In the upgraded Ioffe-Pritchard magnetic trap, the dipole barriers are sent vertically through the trap, along the y-direction. In the horizontal radial direction (x-axis), the laser waist is much larger than the radial extent of the BECs (see figure 2.16 for trap orientations).

Light for the red-detuned dipole beam is provided by a 830 nm diode (Sanyo DL7032-001, 100 mW) and light for the blue-detuned beam comes from a 660 nm diode (Mitsubishi ML101J27, 120 mW). Figure 2.11 shows false-color images of both beams’ profiles along with their fitted cross sections. We extract the beam waists of for the barriers from these fits.

The output power from the dipole barrier diodes are computer controlled by varying the
Figure 2.10: Optical layout for attractive/repulsive dipole barriers on the (a) output and (b) input side of the fiber. In (b), the dashed and dotted boxes indicate setups for the attractive and repulsive dipole barriers, respectively.

Figure 2.11: Profiles for the (a) repulsive (660 nm) and (c) attractive (830 nm) dipole barriers. (b) and (d) are integrated cross sections of (a) and (b) respectively along the x axis. A Gaussian is fitted to these cross sections to extract the beam waists.
diode current through the use of a DAC channel or by using an AOM. In addition to jumping
the barrier on and off, it can also be controlled using a servo allowing for intensity ramps.

2.3.4 Fibers

Light which has been prepared for various purposes (cooling, repump, imaging, etc.) is
guided through 10 m long single mode, nonpolarization-maintaining fibers from Thorlabs
onto the table housing our machine. The fibers spatially filter the light so that the output is
TEM$_{00}$ mode. Using fibers in our set up also ensures that the position of the beams on the
machine table do not change if modifications to the laser systems are made. We typically
obtain a coupling efficiency between 70% to 80% using fiber patch cables with FC/APC
connectors and standard Thorlabs coupling optics.

With variations in temperature and slight changes of position, the birefringent properties
of the fiber can change which, in turn, can cause changes in the polarization of the light
emerging from the fiber. To manage polarization drift issues, we place, in the fiber line
shortly before the fiber output, homebuilt polarization controllers in the form of coils of the
fiber [35, 36]. A single coil of diameter $\sim 6.7$ cm acts as a quarter waveplate for 780 nm
light while two coils of the same diameter act as a half-wave plate. The coils of fiber are
held in place with Delrin squares. These “fiber flaps” are then secured with an aluminum
bracket which allow the flaps to be rotated. Figure 2.12 shows images of the setup along
with approximate dimensions. We find that once the polarization is set using the flaps, the
output of the fiber is reliably consistent over the course of many months. Very rarely do
we find the need to readjust the flaps to correct for polarization drifts. To ensure that the
correct linear polarization comes out of the fiber, polarizing beam splitters are used directly
behind the fiber output coupler. In the case of the MOT light, only about 3 mW of power
(with a total power of $\sim 200$ mW out of the fiber) is lost to the wrong polarization when the
flaps are adjusted correctly.
2.4 Magneto-Optical Trap

The first step in making a BEC begins with the magneto-optical trap (MOT). This is where $^{87}\text{Rb}$ atoms released from atomic source dispensers are captured and pre-cooled to temperatures on the order of tens of $\mu$K. The MOT consists of six beams and a pair of coils in anti-Helmholtz configuration. In our setup, these coils serve two purposes. First, they confine atoms in the MOT and second, they are used to transfer the atoms captured in the MOT to the science trap. Thus, these coils are referred to as both the MOT coils and the transfer coils.

After loading the MOT for 10 s, we perform a CMOT (compressed MOT) stage, molasses, and optical pumping on the pre-cooled atoms to produce a spin polarized sample in one of the two maximally stretched low-field seeking states in the hyperfine ground state manifold of $^{87}\text{Rb}$ (i.e., either the $|F, m_F\rangle = |1, -1\rangle$ or $|2, 2\rangle$ state). For potassium, we prepare the atoms in the $|9/2, 9/2\rangle$ state and use sympathetic cooling with Rb in the $|2, 2\rangle$ state. The atoms are then transferred from the MOT cell to the science cell using a track system in which the MOT/transfer coils physically ferry the atoms across a differential pumping stage. For the transfer, the current in the MOT coils is first jumped to 220 A and then ramped up to about 400 A in $\sim 200$ ms. The atoms travel a distance of about 78.5 cm in less than 2.5 s to the science cell. Upon arrival at the science cell, the atoms are transferred into the
science trap (either the HIP trap or the fully electromagnetic Ioffe-Pritchard trap). In the following sections, details about the individual steps of this scheme are provided.

2.4.1 Atomic Sources

The current machine is equipped with Rb and K ampoules (purchased from Technical Glass) as well as dispensers (from Alvatec) for both atomic species. 50 mg of Rb and 5 mg of enriched $^{40}\text{K}$ are contained in F-type Alvasource getters. The enrichment for $^{40}\text{K}$ is necessary because of its low (0.0117%) natural abundance. When the first of these potassium dispensers were installed and activated by running the required current of 6 A, no potassium atoms were observed in the MOT cell even after running the dispensers for several days. This lead to the tedious and time consuming task of opening up the machine again to install new dispensers and baking out the entire machine again.

A second attempt produced more promising results though when we first activated the Rb dispenser the indium seal of the dispenser exploded, leaving bits of debris lying on the bottom of our MOT cell. According to the manufacturer, this happens occasionally and is due to raising the temperature too slowly when initially attempting to break the indium seal. Luckily, the debris do not interfere with the loading of our MOT. We find that running 3 A of current through the Rb dispensers releases atoms at a constant rate while the K dispensers need about 5 A. When both species of atoms are needed, running the K dispensers at 5.25 A and the Rb dispensers at 2.75 A produces temperatures high enough to release a sufficient amount of atoms for both species (heat from the K dispenser translates to the Rb dispenser fairly efficiently).

It should also be noted that the dispensers were not degassed during the machine bakeout since the dispensers are closed by an indium seal and, due to their high purity, do not need to be degassed. When we first activated the dispensers, there was a jump in the ion pump current due to the release of argon gas, which was contained within the dispensers, when the indium seal was broken. The amount of Ar is actually significant, and since ion pumps do not work well for Ar, opening the seal should best be done with the turbo pump still connected.
Rb dispenser
K dispenser

Figure 2.13: Dispenser mounts machined from UHV copper mounted on a Macor support. Electrical connections are made using UHV copper wire insulated with ceramic beads.

to the system. However, we did not have any problems with activating the dispensers after the bakeout procedure and pumping out the contaminants, and the ion pumps do not detect any gas load from running the dispensers on a daily basis.

The dispensers are mounted on a homemade contraption composed of small UHV grade (OFHC) copper parts all mounted on a Macor boat. UHV copper wire is used to connect the dispenser mount blocks to the electrical feedthroughs. To prevent electrical shorts, the copper wires are insulated with ceramic “fish-spine” beads. Figure 2.13 shows the assembled mounts and dispensers connected to the CF flange electrical feedthrough.

Over time, atoms tend to coat the glass cells. Desorption of the build-up of alkali atoms is activated by shining an array of bright blue LEDs (purchased from Roithner Lasertechnik GmbH) onto the glass cells to promote light induced atomic desorption (LIAD) [37, 38]. The LIAD LED array is also turned on during the MOT loading stage of the atoms to help increase background pressure, and is turned off immediately after the 10 s MOT load. A notable difference in the number of atoms captured is observed. For the science cell, a single blue LED is turned on overnight and whenever the machine is not in operation to prevent build-up of Rb over time.

The Rb and K ampoules are kept in the machine as backup sources. The Rb ampoule
Figure 2.14: Optical layout of (a) the four horizontal MOT beams and repump, (b) the two vertical MOT beams along with the optical pumping beam.

has not yet been broken where as the K ampoule is open, but does not seem to provide sufficient amount of atoms. Unlike the dispensers, the ampoules cannot be turned off nor can the amount of atomic vapor released be controlled to the same degree as the dispensers, though some decree of control can be achieved by changing the temperature of the ampoules and with the use of a valve between the MOT chamber and the ampoules.

2.4.2 MOT Optics

Our MOT uses six cooling beams arranged as shown in figure 2.14. A total of 210 mW is emitted from the fiber prior to being split, providing about 35 mW to each of the six beams. Power in each of the six beams can be balanced by adjusting the half-waveplates before each polarizing beam splitter cube. Each cooling beam is expanded to a useful diameter of $\sim 1.75''$ with telescopes composed of 1” and 3” diameter lenses of focal lengths 150 mm and -25 mm. Quarter-wave plates before the telescopes change the linearly polarized light to circularly polarized light of appropriate handedness for a working MOT. The overlap alignment of the beams is first roughly accomplished with the aid of a paper mask indicating the ideal position of the beams at the surface of the MOT cell. Two small cameras sensitive to infrared
light mounted above and behind the MOT are used to observe the MOT fluorescence on a TV monitor. Once a MOT, even if it is a very small one, can be seen on this monitor, adjustments to the cooling beams can be made to optimize the shape, position and size of the MOT (see below for MOT optimization).

Repump light for rubidium is overlapped with the four horizontal cooling beams, but is absent in the two vertical cooling beams. This does not present any problems as the rubidium MOT does not require very much repump light to function. A total of 7 to 9 mW is split amongst the four repump beams. In the case for potassium, more repump light is needed, thus it is amplified along with the potassium MOT light as can be seen in figure 2.8. A total of $\sim 200$ mW is split into six K MOT/repump beams and is overlapped with the Rb MOT light (see fig. 2.14).

### 2.4.3 MOT Coils

The MOT/transfer coils, arranged in anti-Helmholz configuration, are wound using hollow copper wire with a square outer cross section (ID $0.098''$ round, OD $0.162''$ square from Wolverine, with Dupont 150 FW 019 Kapton 1/2 lapped coating provided by S&W Wire Company). Each coil, with an ID of $\sim 13$ cm, has 6 turns and is 4 turns tall. These coils are water cooled using the same chiller system as for the science trap coils (see fig. 2.18 and section 2.7). During the MOT phase, a current of $\sim 13$ A runs through these coils, but as the atoms are transferred from the MOT cell to the science cell, a maximum of about 390 A is applied. Thus, 4/0 gauge heavy duty welding cable is used to make the connections between the coils and our HP/Agilent power supply (HP 6690A 0-15V 0-440A) operated in constant voltage mode. Fast turn-on/turn-off of the upper and lower transfer coils is provided by a set of 10 MOSFETs mounted on a copper water-cooling block. To reduce the load on the MOSFETs, the voltage is adapted to the maximum currents for the MOT loading and transfer stages.

The transfer coils are attached to a cart which sits on a linear translation stage (from Parker) and driven by a motor from Animatics (SmartMotor SM3440D). This cart system
enables atoms in the MOT cell to be transferred to the science cell a distance of about 78.5 cm away with a precision of a few microns in less than 2.5 s. The motor is controlled via a parallel connection to the control computer with the aid of the included SmartMotor program and triggered via TTL signals.

2.4.4 MOT Optimization

Optical molasses, in which the quadrupole coils are turned off while the cooling beams are kept on, provides us with good indication of the misalignment in the cooling beams relative to each other and to the quadrupole coils. Since the cooling beams by themselves will not trap atoms within the MOT, atoms will slowly diffuse out of the trapping area, creating a “molasses” of atoms. The directional drift of atoms from the trapping region is always clearly visible and thus allows us to make fine adjustments to the position of the MOT coils along the track or tweak the overlap of the cooling beams. Ideally, the molasses should expand isotropically from its position in the quadrupole trap and should be slow enough to be clearly visible on the TV monitor.

Stray magnetic fields originating from, for example, ion pumps in the vicinity of the MOT cell can change the location of the MOT and lead to less efficient transfer to the magnetic trap. To eliminate these stray fields, we use a set of three shim coils placed around the MOT cell along the axes of the cooling beams (see figure 2.2). We adjust the currents through each shim coil by looking at the shape and speed with which atoms expand out of optical molasses along the vertical and horizontal directions. Eliminating all directionally biased or swirling actions of the atoms is not always possible. However, once the optical molasses is such that the atoms expand outwards fairly evenly on all sides, a significant improvement in the number of atoms captured by the transfer coils is observed. We also find that the shim coils need to be readjusted once every few months.

Another indication of how well the MOT is optimized is to make a CMOT and compare its position with that of the regular MOT. In the CMOT, the effects of the cooling and repump beams are reduced by means of detuning their frequencies. The atoms scatter less
photons and the radiation pressure of the MOT is reduced thereby decreasing the spatial extent of the trapped atoms. This gives a more accurate location of the quadrupole trap center. By making a MOT followed by a CMOT, we can fine-tune cooling beam parameters and MOT coil positions to minimize the difference between the CMOT and MOT positions as observed on the TV monitor. To optimize the overlap between the MOT and the magnetic trap, we keep the MOT beams on and jump the current through the coils between a low (MOT) and high value. The cloud of atoms shrink during the high current phase, but no positional shift should be visible.

2.4.5 Optical Pumping

Rubidium – Immediately after the MOT stage, we selectively prepare our atoms to be in either the $|F, m_F\rangle = |1, -1\rangle$ or $|2, 2\rangle$ hyperfine state. Both these states are low-field seeking and therefore magnetically trappable. Optical pumping into either state first begins after a CMOT phase in which the frequencies of the cooling and repump lasers are detuned by $\sim 35$ MHz and $\sim 140$ MHz to the red and blue side of resonance, respectively. The CMOT stage lasts for about 13 ms and prepares the cloud for being loaded into the transfer coils by decreasing the cloud size\(^6\). To obtain a cloud of atoms in the $|1, -1\rangle$ state, the repump light is switched off immediately after the CMOT phase while the MOT beams are kept on for 2.25 ms. Without the repump, the atoms fall to the F=1 manifold where only the $|1, -1\rangle$ atoms are trapped and the rest are lost.

To prepare a cloud of Rb atoms in the $|2, 2\rangle$ state, the CMOT is turned off after 13 ms and the optical pumping coils (see fig. 2.3) are jumped on to provide a quantization axis of a few Gauss. Light resonant with the $F = 2 \rightarrow F' = 2$ transition, and $\sigma^+$ polarized, is then applied for a duration of about 5 ms to optically pump the atoms into the $|2, 2\rangle$ state. The repump light, back at its original frequency since the end of the CMOT phase, is kept on during the optical pumping phase to prevent population of atoms back into the $F = 1$ manifold. Using the $F = 2 \rightarrow F' = 2$ transition has the advantage that the atoms stop

\(^6\)A large cloud size would lead to a large Zeeman energy when jumping on the transfer trap.
cycling, and thus are not unnecessarily heated, when they reach the desired $|2, 2\rangle$ state. This is because there is no upper state in the $F' = 2$ manifold which the $\sigma^+$ light can couple atoms in the $|2, 2\rangle$ state to. Typically, 0.5 mW of power is used for the rubidium optical pumping.

Potassium – We optically pump $^{40}$K atoms into the $|9/2, 9/2\rangle$ state. After the CMOT phase for Rb, we jump the K MOT laser frequency so that it is resonant with the $F = 9/2 \rightarrow F' = 9/2$ transition. This light is sent through the optical pumping fiber in figure 2.8 and $\sigma^+$ polarized at the fiber output end before it is applied to the atoms for 3 ms. Repump light is also present to prevent population of unwanted states.

The state selected atoms are caught in the transfer coils in which the current is ramped up to $\sim 390$ A within 200 ms and moved to the science cell.

2.5 Hybrid Ioffe-Pritchard Trap

Prior to 2009, a hybrid Ioffe-Pritchard (HIP) trap was used to confine $^{87}$Rb atoms in the science cell. The design of this trap closely follows that of reference [39] and [40] in which permanent magnets are used to create a quadrupole field for radial ($x$ and $y$ directions) confinement and electromagnets (pinch coils) for axial ($z$ direction) confinement of the atoms. Another set of (bias) coils in Helmholtz configuration provide a bias field in the $z$-direction.
which opposes the pinch field and can significantly stiffen the trap in the radial direction. Both sets of coils are mounted on a boron nitride form. Since [39] and [40] describes this trap in detail, this thesis will not cover the specifics. Figure 2.15 shows a picture of the trap before and after it was removed from the rest of the machine.

One of the most attractive features of the HIP trap is the advantage of being able to create large (radial) magnetic fields without the use of current sources and water cooling. In our trap, a gradient of about 510 G/cm is produced in the radial direction by the permanent magnets. A current of 13 A in the pinch coils and 7 A in the bias coils flows constantly while the machine is running\(^7\) to prevent temperature fluctuations resulting from current ramps. Any fluctuation in temperature would lead to a change in the evaporation end point as the strength of the permanent magnets is temperature dependent. To keep the electromagnetic coils at a reasonable temperature, the trap is cooled using compressed air supplied by the building. A nozzle placed below and beside the trap (see fig. 2.15) blows air at the trap whenever the machine is running.

### 2.6 Fully Electromagnetic Ioffe-Pritchard Trap

The decision to upgrade the HIP trap to an Ioffe-Pritchard type trap utilizing only electromagnets was made based on two major concerns. First, the HIP trap limits optical access to the atoms. This meant that the addition of an optical dipole trap as well as optical lattices, which have become one of the most commonly used tools in ultra-cold atomic studies, could not be implemented. Second, the HIP trap also utilizes permanent magnets which naturally cannot be switched off. This combined with the limited optical access places severe limitations on the type of experiments that can be performed. For example, experiments in which Feshbach resonances are required were out of reach while experiments with two-component BECs were restricted to magnetically trappable states (e.g. $|1, -1\rangle$ and $|2, 1\rangle$).

Our Ioffe-Pritchard type trap is based on a design from Scott Papp at JILA/University of

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\(^7\)Except during the moment when the transfer coils arrive at the science trap with the atoms, during imaging, and for some experiments after BECs are created.
Figure 2.16: (a) Schematic diagram of our Ioffe-Pritchard type trap showing Ioffe bars (blue), pinch coils (green), bias coils (yellow) and little bias coils (red) along with the science cell. Directions of currents are indicated by white arrows. (b) Picture of the Ioffe-Pritchard type magnetic trap before installation. The ends of the Ioffe bars and tops of the bias coils are visible. Pinch coils are hidden from view. Little bias coils have not been installed yet.

Colorado, details of which can be found in [41]. Figure 2.16 (a) shows a schematic drawing of the trap with its racetrack Ioffe bars (blue), pinch coils (green) and bias coils (yellow) along with current directions indicated by arrows. An additional set of of bias coils (red), referred to as “little bias coils” in the following pages, are wound directly over the large bias coils. This smaller set of bias coils (10 turns each) are wound using 18 gauge Kapton coated copper wire and can be controlled independently from each other for additional magnetic gradients. Figure 2.16 (b) is a picture of the trap prior to being completed.

The Ioffe-Pritchard trap coils (with the exception of the little bias coils) are made from the same hollow Kapton coated copper wire as the transfer coils (see section 2.4.3) to allow for water cooling. The coils of the magnetic trap are fixed in place with a frame shaped from nonmagnetic Garolite (G-11) and titanium rods. Our fully electromagnetic Ioffe-Pritchard trap has been trouble free since it was installed in 2009 with the exception of the occasional coil cleanings and one broken Swagelok connector in the cooling tubes.

Current through the transfer and science cell trap coils is supplied with an HP/Agilent power supply as shown in figure 2.17. IGBTs are used to switch the Ioffe, pinch and bias coils in the upgraded trap. As with the MOSFETs, the IGBTs are mounted on water cooled copper blocks. In addition, a flow of compressed building air is used to cool the connectors of
the IGBTs. The current servos are home-made and designed by Fred Schuetze in the WSU electronics shop. The IGBT servo can jump the Ioffe/pinch coils on within 2.5 ms and off within 50 µs. Large jumps overshoot the desired value by small amounts, but the oscillations die out within 2.5 ms. The bias coils can be jumped on within 600 µs and off within 10 µs.

While the machine is in operation, a current of about 390 A at 7 V is supplied to the science trap, otherwise the HP/Agilent runs a current of 13 A through the MOT coils.

To optimize loading into the science trap, we transfer atoms into the science trap, recapture them in the transfer coils, move the atoms back to the MOT cell, flash on the MOT beams and look at the fluorescence signal from the leftover atoms. The position of the cart at the science cell can be adjusted accordingly to maximize the fluorescence signal.

2.7 Chiller Unit

Temperature control of the electromagnetic coils is an important factor in our setup as heat generated must be dissipated not only in order to prevent damage to the coils and electronics,
but also to prevent drifts in the magnetic fields produced. Slight drifts in the magnetic fields would mean drifts in the magnetic trap bottom which would generate reproducibility problems in creating a BEC. Figure 2.18 shows a diagram of our water chiller system.

A small in-line water filter is used to remove dirt and rust particles from the source water for our chiller system. A small amount of the filtered water is branched off to provide cooling to the MOSFETs and IGBTs in the electrical circuit of the coils while the rest is diverted to the main trapping coils. The source water at $\sim 20^\circ$C is first cooled with water from WSU’s chilled water facilities, which is at a temperature of $\sim 9^\circ$C. The magnetic trap coils as well as the transfer coils are cooled using this water through a closed loop as shown in figure 2.18. We choose a setpoint of $25^\circ$C for the water pushed through the coils at a pressure of 150 psi using a motor from Emerson (Model S55CXPNNA-7672) and pump from Procon (Model 114B240F11BA250). A valve placed directly after the pump allows us to short-circuit the cooling loop of the electromagnetic coils. This is done during the turn-on and turn-off of the pump to avoid pressure pulses running through the coils. In addition to monitoring the flow rate through each coil, we recently implemented an Arduino based temperature alarm to measure the temperature of the Ioffe/pinch and bias coils directly at the trap. During constant operation of the machine, these coils heat up by less than about $10^\circ$C. The reservoir of the electromagnetic coil loops is filled a little over half-full with distilled water. At one point, we used a mixture of filtered tap water and deionized water,
since deionized water alone corrodes the copper coils. While using this mixture, we also had problems with bacteria and algae growth as over time, a slimy build-up would appear in the reservoir tank. Changing from a mixture of tap water and deionized water to pure distilled water alleviated these problems.

Periodic “coil cleansings” are also required to remove calcium deposits to maintain an adequate cooling power to meet the 25°C setpoint. Every 5 to 6 months, an increase in the trap temperature measured by the Arduino indicates the necessity for cleaning. Occasionally, one of the flow switches could also trip and indicate that it is time to clean the coils, but a reduction of cooling power can occur before there is a noticeable decrease in the flow rate. Hence, the temperature measurement is a better indicator. Cleaning the coils was originally done by flushing a mild solution of oxalic acid through the coils for up to six hours with the use of a small electric water pump (Powercool UL7500, 115V). However, we were apprehensive about the effectiveness of the oxalic acid on the copper coils and in the end, switched to using CLR instead. The CLR is added to the water reservoir and pumped through the system for about 5-6 hours. This is followed by a very thorough rinse of the system with tap water before it is filled with fresh distilled water. Each time we clean the coils, we use about 0.4 L (half of a 28 fl oz bottle) of CLR.

2.8 Evaporative Cooling

Radio frequency (RF) signals are used to induce spin flips of the atoms. In combination with the spatially dependent Zeeman shifts due to the magnetic trap, spin flips can be used to selectively remove the hottest atoms from the trap, effecting evaporative cooling. Magnetically trapped atoms in the $|1, -1\rangle$ state are subject to several successive RF ramps decreasing in frequency, the first of which is performed in the quadrupole trap. Five more ramps, each at a different rate, cool the atoms further in the magnetic trap. The trap is then relaxed before a final evaporative RF ramp cools the atoms down to BEC. If we wish to load atoms into the optical dipole trap, we stop the evaporation 200 kHz above the BEC.
transition in the final RF ramp. For our dipole trap beam parameters, a power of about 200 mW corresponds to a trap depth of \( \sim 25 \mu m \) which is deep enough to confine the atoms. A similar, but slightly modified evaporation procedure is used for the \( |2, 2\rangle \) state.

The RF radiation is generated using a Tektronix Arbitrary/Function Generator (AFG 3101) controlled through a GPIB interface with LabVIEW. The signal from the AFG is amplified using a broadband RF power amplifier (ENI 310L) before it is sent to the atoms. The RF coil is a single loop 2 cm in diameter made from 18 gauge Kapton insulated copper wire. The coil is positioned close to the atoms and is oriented such that its axis is perpendicular to the bias field (thus it produces \( \sigma \)-polarized fields). center of the trap.

### 2.9 Optical Dipole Trap

The optical dipole trap used in our experiments originates from a 20 W IPG Photonics fiber laser at 1064 nm. This laser also provides light for optical lattice beams (to be discussed in a future thesis) and is split accordingly with half-wave plates and polarizing beam cubes as shown in figure 2.19. High power fibers from OZ Optics are used to guide light from the laser table to the machine table where a custom cage system is used to mount the
output collimating lenses. The output end of the OZ Optics fiber is AFC (angled flat polish) terminated and we couple this to an FC fiber adapter. Several versions of the output coupler were tested using lenses and lens tube mounts from different companies before we settled on the current setup. We note that astigmatism, mainly originating from misalignment of the lenses, was the main source of trouble in the setup of the dipole trap. In the final arrangement, the beam is collimated with a combination of a small focal length lens (Thorlabs C230TME-C, NA=0.55) and a f=75 mm achromatic lens (Thorlabs AC243-075-C) to a measured $1/e^2$ radius of about 2.5 mm. We found it extremely helpful to place the Pixelfly camera in the (attenuated) beam path after each lens for beam profiling as care must be taken to minimize the astigmatism while assembling the lenses. Our dipole beam has an astigmatism less than 50 µm.

A laser line beam splitter cube (Newport) sets the polarization and a pick-off window (Newport) sends a small fraction of the light to a photodiode for intensity control. The entire output assembly, plus the mirror used to direct the beam towards the science cell, is mounted on a large aluminum block (see fig. 2.20) to help reduce drift and vibrational noise. Using a 250 mm CVI/Melles Griot achromat mounted on an xyz translation stage (which is also mounted on a sturdy block of aluminum), the collimated beam is then focused down to a beam waist of 26 µm at the center of the magnetic trap. A minimum power of $\sim 19$ mW, measured in front of the science cell, is needed to hold a BEC against gravity.

Atoms evaporatively cooled to BEC (or precooled close to the BEC transition) in the fully magnetic Ioffe-Pritchard type trap can then be loaded into the optical dipole trap. We have two choices for the initial state: either the $|1, -1\rangle$ or the $|2, 2\rangle$. To populate states other than these, an adiabatic rapid transfer can be used. Initial rough alignment of the dipole trap was performed by overlapping the dipole beam with the imaging beam along the z-axis (BEC axis). We can then check the dipole beam position by taking in-trap images, first of atoms in only the magnetic trap, and second, atoms in a combined magnetic and dipole trap where the dipole power is ramped up to its full strength. This is done along both the x- and y-directions. The differences in the position of the atoms in these two situations can
then be minimized by moving dipole focus accordingly. If no difference in the positions of atoms in the magnetic and magnetic plus dipole trap is observed, it is sometimes helpful to load a thermal cloud into the traps as the spatial extent of “hot” atoms is much greater than that of a BEC is so gives us a greater chance of locating the dipole beam. Adjusting the location of the dipole beam focus is easily done with the translation stage on which the focusing achromat sits. Adjusting the tilt of the beam is done by placing shims of varying thicknesses under the focusing achromat mount.

We typically precool atoms in the magnetic trap before loading them into the dipole trap. A final evaporative cooling phase in the dipole trap, performed by lowering the laser power from 400 mW to 25 mW in 20.5 s in three consecutive ramps, ensures little or no thermal atoms in the final BECs containing about $1 \times 10^6$ atoms in the $|1, -1\rangle$ state. Typical trapping frequencies at the evaporation endpoint are $(\omega_x, \omega_y, \omega_z) = 2\pi \times (170, 120, 1.2)$ Hz, giving an aspect ratio of about 170 (compared to $\sim 60$ in the magnetic trap).
Figure 2.21: Imaging the BEC or DFG is accomplished using a simple microscope setup. Along our main imaging axis in the upgraded science trap, the objective is a $f=40$ mm gradient-index lens from LightPath Technologies (GPX-15-40) and the second lens is a Ø2", $f=750$ mm achromat from Thorlabs. The magnification along this axis is $\sim 17$. The objective used for the HIP trap was a GPX-10-10 which gave a magnification of 19.

2.10 Imaging System

Standard absorption imaging [42] is used to collect information about the physics of our system. The process is the same for both atomic species and begins with sending a beam of collimated light tuned to a specific atomic transition through the science cell to probe the atoms. The atoms absorb the probe light to form a shadow in the beam. This shadow is imaged onto a CCD camera using a pair of lenses, shown schematically in figure 2.21, thereby recording the density distribution of the atoms. The amount of light absorbed out of the probe beam depends on the beam intensity, pulse duration, and density of atoms. We pulse on the probe beam for $10 \mu s$ and the most useful images are those in which the clouds have been expanded so that it is not completely opaque to the probe beam. In the original HIP trap, anti-trapped expansion was used because of the permanent magnets. In the upgraded fully electromagnetic IP trap, free expansion does the trick. In some cases, a very short expansion times can be useful in extracting the “in-trap” position of the clouds. Two more images are captured after the BEC is destroyed by absorbing the probe light: one with just the probe beam and one with no light. These three frames are then used to compute the optical density of the cloud (see section 3.2).

Imaging light for rubidium is provided by a commercial Vortex ECDL at 780 nm (see section 2.3). Figure 2.22 shows a schematic of the optical layout for our imaging beam on the fiber input side. To image atoms in the $|1, -1\rangle$ state, we typically lock the laser to the $F = 1 \rightarrow F' = 1$ transition, and to image atoms in the $|2, 2\rangle$ state, the laser is locked to the $F = 2 \rightarrow F' = 3$ transition (see figure 2.7). The first of two AOMs shifts the laser
frequency up by $\sim 120$ MHz and the second shifts it back down near the locking frequency. This arrangement allows for precise frequency tuning in order to minimize lensing effects.

Anti-trapped Expansion Imaging [39] — Imaging in the HIP trap poses special difficulties because the radial gradients generated by the permanent magnets cannot be turned off. Hence we image the atoms in the presence of a large bias field which adds in quadrature to the radial gradients. This reduces the magnetic field variation across the cloud to a degree acceptable for imaging. To obtain images of Rb atoms in the in the permanent quadrupole field of the HIP trap, we first transfer atoms from the $|1, -1\rangle$ state to the high-field seeking $|2, -2\rangle$ state. This is done via an adiabatic rapid transfer (ARP) where a fixed microwave field is applied to the atoms while, at the same time, slowly ramping$^8$ the magnetic fields produced by the bias coils through the $|1, -1\rangle \rightarrow |2, -2\rangle$ resonance. Our microwave source comes from a 6.81 GHz stable reference (Nexyn Corp. NXPLOS-I-0681-02235) mixed with RF frequencies using a single side band Pulsar Microwave mixer (IROH-H5-458). The signal is amplified (using a 15 W amplifier from Microwave Solutions, Inc., Model No. MSH-5727901) then sent to the atoms via a waveguide (J40-3C from Microwave Engineering Corp.). After the atoms have been transferred to the $|2, -2\rangle$ state, a 100 G bias field is

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$^8$Slow compared to the Rabi cycling.
applied so that the $|2, -2\rangle \rightarrow |3', -3\rangle$ cycling transition is isolated for the imaging light. Figure 2.23 indicates the energy splittings of the involved states in a 100 G field. For most purposes, we wait 2 to 3 ms after the ARP before applying a 10 $\mu$s imaging pulse.

**Free Expansion Imaging** – Imaging in the upgraded Ioffe-Pritchard trap is somewhat simpler since all magnetic field and gradients originating from the trap can be turned off within 50 $\mu$s. Typical free expansion of the atoms under the influence of gravity between 7 to 10 ms is enough to clearly reveal structures such as solitons otherwise unidentifiable in in-trap images. The free expansion dynamics of a cloud of atoms released from harmonic trapping potentials is a well studied subject, details of which can be found, for example, in [42] and [43].

Potassium atoms are imaged on the $F = 9/2 \rightarrow F' = 11/2$ transition (see section 2.3.2).

**Combining K and Rb imaging** – On the output side of the imaging fibers, both for Rb and K, light is split into three beams for probing atoms along each of three orthogonal directions. The output platform is shown in figure 2.24 where the paths of the three imaging beams are as indicated. The relative powers in each branch are adjusted using a combination of two waveplates and a PBS. The first waveplate only affects rubidium light ($\lambda/2$ at 780 nm and was specially made by Lens Optics) and the second waveplate affects both wavelengths of

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**Figure 2.23:** (a) Energy level diagram for the D2 lines of $^{87}$Rb. The MOT and repump transitions are indicated. (b) Zeeman energy shifts of selected levels in $^{87}$Rb in a 100 G bias field. Note that this diagram is not to scale.
light. Our main imaging axis is along the $y$-direction (the long axis of the machine). Imaging along the $z$-direction (the long axis of the BEC) does not yield useful information about the atomic density distribution of a cloud because it is opaque along this axis. However, it is very useful when the spatial position of a BEC is needed for alignment purposes, for example. The rubidium imaging beam has a measured diameter of about 2 mm (compare to expanded BEC width of 600 $\mu$m) and a power of 500 $\mu$W coupled out of the fiber. About 250 $\mu$W is used in the main imaging branch.

### 2.10.1 Cameras

There are two cameras currently in use. The primary camera is a Princeton Instruments PIXIS 1024 BR which is back illuminated and uses deep depletion CCD technology for QE better than 90%. It also is capable of kinetics mode. This camera has a $1024 \times 1024$ sensor.
array with a $13 \times 13 \mu m$ pixel size and is cooled to -70°C to reduce pixel noise. No fringes produced by the camera alone are visible when imaging with 780 nm light. We use the PIXIS to image along the horizontal radial axis as well as in the BEC axial direction. A second camera, the Pixelfly qe ($1392 \times 1024$ pixels with $6.45 \mu m$ square pixels) from Cooke Corporation, is used to image along the vertical direction (y-axis). To avoid imaging fringes possibly produced by the camera, the CCD window was removed. The Pixelfly is not cooled and thus suffers from pixel noise, however it is significantly cheaper than the PIXIS. In comparison, the cooled PIXIS CCD chip provides noticeably better pixel noise. However, the small size of the Pixelfly allows for flexibility and portability so that we can move the camera to almost any convenient position. This is especially handy for alignment of the optical dipole trap and lattice beams as well as for beam profiling. Another advantage provided by the Pixelfly is that it has an interline transfer CCD, making it possible to capture two successive images very quickly in time.

A $2' \times 2'$ optical platform machined from Garolite placed on four aluminum supports sits above the trap and houses the Pixelfly as well as optics for the vertical optical lattice beam.

2.11 Computer Control

Experimenting with BECs in the laboratory is a multi step process requiring the precise timing of commands down to microsecond resolution. Each step of the process is computer controlled using two PCs running Windows XP. A typical day of experimentation in the lab means that we spend most of our time in front of the computers. One computer is used exclusively to control the sequence of commands ultimately leading to the production of a condensate and various experiments. The second computer is responsible for capturing, storing and analyzing images of the atoms. Communication between the computers and various parts of the BEC machine is accomplished via digital, analogue, serial, and GPIB signals. Prior to the machine upgrade, a National Instruments PCI card (NI PCI-6229) along with a DIO-64 board from Viewpoint Systems provided the necessary digital and analogue
inputs and outputs for our computer control system. A second DIO-64 board was installed during the upgrade in anticipation of new additions to the machine. The digital ports are used for controlling devices such as shutters, switches, LIAD lights, etc. and the analogue ports are used for trap coils and other voltage/current controlled equipment. The PCI-6229 is synced to the DIO-64 board via its programmable function interface as a trigger and it is mostly used to monitor the MOT fluorescence. The main machine control is effected using the DIO-64 boards. Each DIO-64 board has 64 digital channels, 44 of which are used directly as TTL outputs. Of the remaining 20 TTLs, 16 are used as the set value for homebuilt 16 bit DAC boards, 3 are used to multiplex between 8 different DAC boards, and the last TTL output serves as the trigger line for the DACs. A homebuilt optocoupler board protects all digital lines and keeps electrical noise from the computer away from other electronics. RC filters are occasionally used for the DAC channels to reduce switching glitches. Finally, our computer system also includes a digital logic box containing line drivers, OR and AND chips as well as inverters.

Figure 2.25(a) shows the inputs and outputs to the control computer and figure 2.25(b) shows how the imaging computer communicates with the cameras. Several of our frequency generators (such as the AFG RF generator, two Stanford SRS, etc.) are programmed via GPIB, triggered by TTL outputs, and referenced to a 10 MHz oscillator that is disciplined by a GPS receiver.

The main software used to handle the sequence of experimental events is LabVIEW.
However, we find LabVIEW’s graphical programming environment to be inefficient when it comes to making small and quick modifications to the program. Therefore, we use a program written in C++ to generate a “run script” read by LabVIEW and a timing sequence is loaded into LabVIEW prior to each experimental run. All machine programming is thus done in C++.

Images from the Princeton Instruments camera are captured using WinView while those from the Pixelfly are captured using PCO drivers (which came with the camera) and software written by an undergraduate student working in our lab at the time. The captured images can then be loaded into Winview for analysis using software (which interfaces with Winview) written in large part by Paul Haljan who was working at JILA at the time. We originally purchased LabVIEW drivers for the Princeton Instruments camera which however turned out to be faulty/insufficient, so we resorted to Winview/Visual Basic programming.
Chapter 3

Data Analysis

3.1 Introduction

In this chapter we present a discussion of how useful information about an experiment is extracted from the raw images recorded by our two CCD cameras. Images collected of the condensates, DFGs, and/or thermal clouds provide the only window into which we can look and infer useful information about the atoms. Thus, in addition to having an excellent imaging system, it is equally important to be able to analyze the images consistently and accurately.

One of the most basic and widely used imaging techniques is absorption imaging in which atoms are probed with on-resonant light and a record of the shadow cast by the atoms is imaged on to a CCD chip. This technique is destructive in nature and requires that a BEC be expanded prior to imaging due to problems encountered with the high atomic density of a trapped BEC. While in trap, a BEC is completely opaque to on-resonant light, and slightly detuning the imaging light from resonance would lead to a strong lensing effect caused by the BEC. The theory of how an ultracold cloud of atoms behave as it is released from harmonic traps is well studied and characterized, see for example, [42, 43]. Thus, in most cases, all one has to do is apply the relevant equations using parameters specific to one’s system to extract information about the atoms. In the next few sections, we present the relevant equations
and techniques applicable to analysis of the experiments discussed in chapters 4 and 5.

### 3.2 Image Analysis

Our imaging technique provides us with absorption images of the atoms under study. Each acquired image consists of three frames, a shadow frame, a light frame and a dark frame. In the shadow frame, the intensity of the shadow cast by the atoms is captured. The light frame consists of only the imaging light intensity with no atoms while the dark frame is a record of the background light intensity without the imaging light or atoms present\(^1\). To avoid saturating the imaging transition, the imaging light intensity should be chosen to be sufficiently low. From the three captured frames, the optical depth (O.D.) of each captured pixel is calculated using

\[
\frac{I}{I_0} = e^{-\int \sigma n dx} = e^{\text{O.D.}}. \tag{3.1}
\]

The O.D. is the integrated optical density \(\sigma n\) along the line of sight, integrated through the entire cloud. Rearranging the above gives

\[
\text{O.D.} = \ln \left( \frac{I_0}{I} \right) = \ln \left( \frac{I_{\text{light}} - I_{\text{dark}}}{I_{\text{shadow}} - I_{\text{dark}}} \right) \tag{3.2}
\]

where \(I_0\) is the intensity before entering the cloud and \(I\) is the intensity upon leaving the cloud.

In most cases, we image the atoms after letting the cloud expand for various amounts of time. In the case of the HIP trap, anti-trapped expansion is used (see section 2.10) and in the Ioffe-Pritchard type magnetic trap, the clouds are allowed to expand freely prior to imaging.

Since the absorption images are related to the atom density integrated along the line of sight, useful parameters can be extracted by comparing our O.D. images with two-

\(^1\)The camera electronics may be designed to intentionally produce a non-zero (but uniform) pixel background for zero light due to technical reasons.
dimensional theoretical density distributions. Uncondensed atoms in a thermal cloud can be fitted using a Gaussian while a BEC has a density distribution which can be approximated by the Thomas-Fermi approximation. In this thesis, the atoms are condensed with a high condensate fraction and large atom numbers so that the Thomas Fermi approximation can often be reliably applied.

Integrating through the BEC along the imaging direction, the O.D. in the Thomas-Fermi approximation is given by [42]

\[
\text{O.D.} = |\text{Peak O.D.}| \text{MAX} \left[ \left( 1 - \frac{(x - x_c)^2}{R_{x,TF}^2} - \frac{(y - y_c)^2}{R_{y,TF}^2} \right)^{3/2} , 0 \right] \tag{3.3}
\]

where \(x_c\) and \(y_c\) are the centers of the cloud along the \(x\) and \(y\) directions, and \(R_{x,TF}\) and \(R_{y,TF}\) are the radii of the cloud along their respective directions. The exponent \(3/2\), which differs from the exponent \(2\) found in the Thomas-Fermi 3D density in a harmonic trap, is due to the integration along the line of sight.

For thermal clouds, we fit our captured images to a Gaussian distribution given by [42]

\[
\text{O.D.}_{\text{Gauss}} = |\text{Peak O.D.}| \exp \left( -\frac{(x - x_c)^2}{\sigma_x^2} - \frac{(y - y_c)^2}{\sigma_y^2} \right) \tag{3.4}
\]

where \(\sigma_x\) and \(\sigma_y\) are the widths in the \(x\) and \(y\) directions.

A bimodal distribution takes into account the thermal cloud density in addition to the condensed atoms. From the fits of the O.D. images, the position and widths of the clouds can be directly extracted.

For various reasons, it may also be useful to be able to determine the widths of the clouds while it is in-trap. Because the in-trap density is much too high to be imaged properly using absorption, one must extrapolate the in-trap widths from expansion images (see section 3.5) or resort to other imaging techniques such as phase-contrast or dark-field imaging.
3.3 Magnification Calibration

The parameters extracted from the density fits given in the previous section are typically in units of camera pixels. Thus, it is necessary to convert the fitted values to meaningful units before they can be used. We do this by determining the magnification, $M$, of our imaging system as follows.

In the fully magnetic Ioffe-Pritchard trap, $M$ is determined by turning off all external trapping potentials and letting the cloud fall freely under the influence of gravity for variable amounts of time. Due to the symmetric arrangement of the coils in our science trap, no noticeable momentum is imparted on the atoms by the turn-off, and the atoms start falling according to equation 3.5. The cloud’s position in the direction of gravity is plotted against the expansion time, $t - t_0$, and using a fit of the form

$$y = y_0 + \frac{g}{2M} (t - t_0)^2$$

where $g$ is the acceleration due to gravity, the magnification, $M$, is extracted from the data. Figure 3.1 shows one such measurement in the Ioffe-Pritchard magnetic trap using an imaging lens combination of $f = 40 \text{ mm}$ and $f = 750 \text{ mm}$, giving a magnification of $0.75 \mu\text{m}/\text{pixel}$.

In the HIP trap, we arped atoms to the $|2, 0\rangle$ state and exploited the fact that the $|1, 0\rangle$ state is weakly bound due to quadratic Zeeman shifts, leading to large dipole oscillations in the vertical direction from which the imaging magnification can be inferred.

3.4 Trapping Frequency Measurements

Trapping frequencies in the HIP trap were measured by exciting dipole oscillations using various coils. Radial dipole oscillations were induced by pulsing on the centering coil mounted on top of the trap while axial oscillations were induced by pulsing on the pinch coils (see fig. 2.15). A typical trapping frequency measurement (fig. 3.2) involves plotting out the positions of the BEC as a function of time for at least one full oscillation to begin with and...
several points near the crossings at later times to more precisely determine the frequency. However, data at the turning points are taken as well to check that the oscillations are undamped. We keep the oscillation amplitude as small as possible to avoid probing the anharmonic regions of the trap.

In figure 3.2, we show a typical plot of the BEC position as a function of time from which we fit a sinusoidal curve in order to extract the frequency and amplitude.

Trapping frequency measurements in the Ioffe-Pritchard magnetic trap are measured similarly. In the vertical radial direction, we excite a dipole oscillation by jumping on a centering coil. In the horizontal radial direction, a small coil wrapped around a lens tube...
(which holds the first imaging lens) can be used to create magnetic gradients for exciting dipole oscillations. The most difficult trapping frequency measurement is that in the axial direction. Here, we turn on one of the small bias coils during the evaporative cooling stage and induce dipole oscillations in the BEC by suddenly turning off the small bias coil after the final evaporative cooling ramp. We typically run just enough current through the various coils to excite an oscillation with an amplitude of about 8 $\mu m$. Large amplitude oscillations will probe anharmonic regions of the trap while small amplitude oscillations tend to be lost in the shot-to-shot noise of our system. Both the vertical radial and axial measurements are imaged using the x-direction (machine axis) with the Princeton Instruments camera while the horizontal radial measurement is imaged along the y-direction using the small Pixelfly.

To determine the vertical radial ($y$-axis) trapping frequency in our single beam optical dipole trap, we simply increase the beam intensity briefly to induce a dipole mode in the vertical radial direction. In doing so, we also excite the quadrupole (breathing) mode. However this does not affect the trapping frequency measurements since these two modes are not coupled. Trapping frequencies along the $x$– and $y$–directions are measured with the aid of small electromagnetic coils which are used to slightly shift the center position of the trapped atoms during the final evaporative cooling ramp. Dipole oscillations are induced when these coils are jumped off after forming a BEC.

### 3.5 Thermodynamic Quantities

We make estimates of the condensate chemical potential and atom number based on the assumption that 1) our condensates contain a large number of atoms and 2) the axial extent of the condensate changes very little upon expansion out of the magnetic and dipole traps, which is justified below.

In the Thomas-Fermi approximation for a condensed cloud with large atom number and repulsive interactions, the chemical potential at the edge of the cloud is equal to
\[ \mu = \frac{1}{2} m \omega_{ax}^2 R_{ax}^2 \]  

(3.6)

where \( m \) is the particle mass, \( \omega_{ax} \) is the trapping frequency in the axial direction, and \( R_{ax} \) is the axial Thomas-Fermi radius of the condensate. The above equation follows from neglecting the kinetic energy, \( p^2/2m \), and setting \( \mu = \frac{1}{2} m \omega^2 z^2 + \frac{4\pi \hbar^2 a}{m} |\phi(z)|^2 \). In this approximation, the number of atoms in the condensate is given by

\[ N = \left( \frac{2\mu}{\bar{\omega}} \right)^{5/2} \frac{a_{ho}}{15a_s} \]  

(3.7)

where \( \bar{\omega} = (\omega_{radial} \omega_{axial})^{1/3} \) is the geometric mean of the trapping frequencies and \( a_{ho} = \sqrt{\hbar/m \bar{\omega}} \) is the harmonic oscillator length. Since for the short expansion times of our experiments (3 to 8 ms), the size of the cloud changes very little along this weakly trapped direction (\( \omega_z \approx 2\pi \times 7 \) Hz), the in-trap extent \( R_{ax} \) is approximately unchanged by the expansion. The expansion in the axial direction of the cloud begins to be noticeable after long expansion times on the order of \( \omega_{radial}/\omega_{axial}^2 \).

Along the radial direction, the time evolution of a freely expanding condensate can be determined using a semi-classical treatment of the time dependent Gross-Pitaevskii equation. If all trapping fields are suddenly turned off and the cloud is allowed to expand freely for time \( t_{tof} \), the time-of-flight widths of the condensate in the radial and axial directions are given by [42]

\[ \rho(t_{tof}) = \rho(0) \sqrt{1 + (\omega_\rho t_{tof})^2} \]  

(3.8)

and

\[ z(t_{tof}) = \rho(0) \left( \frac{\omega_\rho}{\omega_z} \right) \left[ 1 + \left( \frac{\omega_z}{\omega_\rho} \right)^2 \left( \omega_\rho t_{tof} \arctan(\omega_\rho t_{tof}) - \ln \sqrt{1 + (\omega_\rho t_{tof})^2} \right) \right] \]  

(3.9)

As confirmation, typical trapping frequencies of \( \omega_z = 2\pi \times 11.72 \) Hz and short \( t_{tof} = 8 \) ms lead to \( z(t_{tof})/z(0) \approx 1 \), justifying the use of equations 3.6 and 3.7. In the case of our optical
dipole trap, in which the two radial trapping frequencies are not exactly the same, the slow axial expansion still holds. However, in the case of low atom numbers in a highly elongated trap, the Thomas-Fermi approximation may not be applicable in the radial direction.

For an axially symmetric trapping potential such as that of our HIP trap, where $\omega_x = \omega_y = \omega_r >> \omega_{axial}$, the time-of-flight widths of the condensate rescales by a factor which can be derived by solving the scaling equations [39].

In the case of a degenerate Fermi gas (DFG), where atoms fill the available energy states from the lowest up, the density profile at $T=0$ is

$$n_F(r) = \frac{1}{6\pi^2} \left( \frac{2m}{\hbar^2} \right)^{3/2} \left[ E_F - V(r) \right]^{3/2} \tag{3.10}$$

where the Fermi energy, $E_F$, is the energy of the highest occupied state. For a harmonic trapping potential with trapping frequencies $\omega_i$, in the $i = x, y, z$ directions,

$$V(x, y, z) = \frac{1}{2} \left( \omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2 \right), \tag{3.11}$$

the number of particles in a DFG is given by

$$N = \int d^3r n_F(r) = \frac{1}{6} \left( \frac{E_F}{\hbar\omega} \right)^3, \tag{3.12}$$

and the Fermi temperature is

$$T_F = \frac{E_F}{k_B} = \frac{\hbar\omega}{k_B} (6N)^{1/3}. \tag{3.13}$$

The zero-temperature density profile becomes

$$n_F(r) = \frac{8}{\pi^2} \frac{N}{R_{F_x} R_{F_y} R_{F_z}} \left[ \text{MAX} \left( 1 - \sum \frac{(x_i - x_c)^2}{R_{F_i}^2}, 0 \right) \right]^{3/2} \tag{3.14}$$

with Fermi radii of the cloud $R_{F_x,y,z} = \sqrt{\frac{2E_F}{m \omega_{x,y,z}^2}}$.

In the case of a “hot” non-degenerate thermal cloud of fermions, the density profile is Gaussian in shape with a size characterized by
\[ \sigma_{F_i} = \sqrt{\frac{2k_BT}{m\omega_i^2}} \]  
(3.15)

and number of atoms

\[ N_{th} = -\left(\frac{k_BT}{\hbar\omega}\right)^3 \text{Li}_3(-\zeta). \]  
(3.16)

Here, \( \text{Li}_n(x) \) is the n-th order Polylogarithm function and the quantity \( \zeta = e^{\mu/k_BT} \) is the fugacity. Combining equations 3.12 and 3.16 gives an expression for the “degeneracy” of a cloud

\[ \frac{T}{T_F} = \left[-6\text{Li}_3(\zeta)\right]^{1/3}. \]  
(3.17)

Images of our DFG are fit to the function

\[ \text{O.D.} = A \frac{\text{Li}_2\left(-\zeta \exp\left[\ln \zeta - \frac{(x-x_c)^2}{R_{Fx}^2} - \frac{(y-y_c)^2}{R_{Fy}^2}\right]\right)}{\text{Li}_2(-\zeta)}. \]  
(3.18)

The temperature of the cloud can be extracted from the fit using

\[ k_BT = \frac{1}{2}m\omega_i^2 \frac{R_{F_i}^2}{\sqrt{1 + \omega_i^2t_{tof}}} \frac{\text{Li}_0(\zeta)}{\text{Li}_1(\zeta)}. \]  
(3.19)
Chapter 4

Dispersive Hydrodynamics via the Merging and Splitting of One-Component BECs

4.1 Overview

Dispersive shock waves (DSWs) have generated interest among many diverse areas of physics. First studied in water and plasma wave dynamics [44, 45], DSWs have also been investigated in other areas where dispersive hydrodynamic behavior is possible including nonlinear optics [46], electronic liquids [47], and ultracold quantum gases [48, 49, 50, 51, 52, 53]. In the experiments described here, we investigate dispersive hydrodynamics in BECs via the merging and splitting of condensates in the nonadiabatic, high-density regime. We realize several important prototypical situations which have been discussed in previous theoretical studies [48, 49, 50, 51, 52].

We begin our discussion in section 4.2 by outlining the theoretical framework with which DSWs in BECs can be modeled. Starting with the Gross-Pitaevskii equation, we summarize its well-known reformulation in the hydrodynamic form [54] to gain insight into how dis-

\footnote{This chapter is based on our publication Phys. Rev. Lett. 101, 170404 (2008).}
persive dynamics can arise. We then summarize, following [55], the 1D dimensionless NLS equation in the small dispersion limit. In section 4.3, we describe our experimental procedure and results. After merging two BECs, the subject of section 4.3.1, we observe the formation of a soliton train. For low enough atom numbers, the soliton train is uniform, as predicted for a one-dimensional situation [48, 56, 57]. For higher atom numbers, a high-density bulge emerges, and numerical simulations\(^2\) suggest that this bulge consists of many vortex rings due to a transverse instability of the soliton train. A precise understanding of the merging dynamics is also essential from a technological point of view. For example, merging processes are fundamental operations in atom interferometers [58, 59, 60] and in the creation of a “continuous BEC” [61] where a condensate is continuously replenished by newly condensed atoms. Furthermore, vortex formation during the merging of multiple BECs has been used as a tool to investigate the relative phases between BECs [62, 63]. In section 4.3.2, we find that splitting a BEC with a repulsive barrier can also lead to DSWs, and we observe a transition from propagating sound waves to DSWs when a sufficiently strong barrier is used. Finally, in section 4.3.3, we find shock dynamics in yet a different setting, namely, when a high-density region in a BEC is suddenly released and allowed to spread into a surrounding background of condensed atoms. Our results complement previous experiments that considered either very narrow initial gaps in a BEC, produced by a stopped-light technique [53, 64], or blast pulses in rotating [65] and nonrotating [52] cylindrical geometries. We give concluding remarks and an outlook regarding the dynamics of splitting an ultracold gas of fermions in section 4.4.

### 4.2 The Gross-Pitaevskii Equation

The theoretical foundation for the study of DSWs in BECs is the small dispersion limit of the one-dimensional nonlinear Schrödinger (NLS) equation that was first studied in [66, 67] and later in many works including [48, 49, 50, 51, 52]. More generally, the three-dimensional NLS equation with a linear potential and small dispersion, also known as the Gross-Pitaevskii

\(^2\)provided by Mark Hoefer
(GP) equation, describes an interacting BEC that can give rise to shock dynamics [52].

The time-dependent GP equation governs the evolution of the macroscopic wave function of a condensate with large atom number at, or very near, zero temperature. Effective interactions between the atoms, which can be either attractive or repulsive, are accounted for using a mean-field approach and give rise to non-linearity in the system. For a condensate whose wave function is given by $\Psi({\bf r})$, where $\bf r = (x,y,z)$ is the spatial coordinate, the time-dependent GP equation is

$$i\hbar \frac{\partial \Psi({\bf r},t)}{\partial t} = \left[-\frac{\hbar^2}{2m} \nabla^2 + V_{\text{ext}}({\bf r}) + g|\Psi({\bf r},t)|^2 \right] \Psi({\bf r},t) \quad (4.1)$$

with an external harmonic trapping potential $V_{\text{ext}}({\bf r})$ given by

$$V_{\text{ext}}(x,y,z) = \frac{1}{2} m \left( \omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2 \right) \quad (4.2)$$

where the $\omega_i$ are the trapping frequencies in the $i = x, y$, and $z$ directions. The coefficient $g = 4\pi\hbar^2 a_s N/m$ in the non-linear term of equation 4.1 describes the mean-field interaction of the atoms and the s-wave scattering length $a_s$ is negative for attractive interactions and positive for repulsive interactions.

To see how dispersive dynamics can arise from the GP equation, it is helpful to reformulate equation 4.1 as a set of hydrodynamic equations [54] which can be compared to the classical Navier-Stokes equation. The usual approach is to begin by writing

$$\Psi({\bf r},t) = \sqrt{n({\bf r},t)} \ e^{i\varphi({\bf r})} \quad (4.3)$$

so that the density is given by $n = |\Psi|^2$, and the velocity $\bf v(r)$ is defined as the gradient of the phase, $\varphi({\bf r})$,

$$\bf v(r) = \frac{\hbar}{m} \nabla \varphi({\bf r}). \quad (4.4)$$

Using equations 4.3 and 4.4, and separating the real and imaginary parts of the resulting
equation, one can show \[54\] that the GP equation can be written as a set of two equations

\begin{align}
\frac{\partial n}{\partial t} + \nabla \cdot (n \mathbf{v}) &= 0 \quad (4.5a) \\
\frac{\partial \mathbf{v}}{\partial t} &= -\frac{1}{mn} \nabla p - \nabla \frac{v^2}{2} + \frac{1}{m} \nabla \left( \frac{\hbar^2}{2m \sqrt{n}} \nabla^2 \sqrt{n} \right) - \frac{1}{m} \nabla V, \quad (4.5b)
\end{align}

with \(p\) being the pressure of the condensate and \(V\) the external potential. Equation 4.5a is a continuity equation for the particle density \(n\), instead of a fluid mass density as in the case for classical hydrodynamics. Equation 4.5b can be compared with the classical Navier-Stokes equation given by,

\[
\frac{\partial \mathbf{v}}{\partial t} - \mathbf{v} \times (\nabla \times \mathbf{v}) = -\frac{1}{mn} \nabla p - \nabla \frac{v^2}{2} - \frac{1}{m} \nabla V + \eta \nabla^2 \mathbf{v}. \quad (4.6)
\]

The second term on the left of equation 4.6 vanishes in the hydrodynamic formulation of the GP equation since \(\nabla \times \nabla \phi = 0\) for a BEC (the condensate is irrotational). The third term on the right of equation 4.5b, missing in equation 4.6, is referred to as the “quantum pressure” term and is responsible for the dispersive dynamics arising in quantum hydrodynamics (as opposed to the dissipative term, \(\eta \nabla^2 \mathbf{v}\), of the Navier-Stokes equation).

In collaborated theoretical work \[55\] performed by Mark Hoefer, the 1D non-linear Schrödinger equation is used to model the dynamics of our system. Starting with equation 4.1 and scaling using the following relations,

\begin{align}
t' &= \omega_\perp t \quad (4.7a) \\
x' &= \frac{x}{l} \quad (4.7b) \\
\Psi' &= l^{3/2} \Psi \quad (4.7c) \\
l &= \left( \frac{4\pi \hbar^2 |a_s| N}{m^2 \omega_\perp^2} \right)^{1/5} \quad (4.7d)
\end{align}

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one arrives at the dimensionless form of the NLS

$$i\varepsilon \frac{\partial \Psi}{\partial t} = -\frac{\varepsilon^2}{2} \nabla^2 \Psi + V_{\text{ext}} \Psi + |\Psi|^2 \Psi \quad (4.8)$$

where $0 < \varepsilon \ll 1$ is called the small dispersion parameter.

### 4.3 DSW Experiment

In the following, we investigate shock waves in a one-component BEC created by merging or splitting BECs or releasing atoms into a nonzero background. Three different situations are realized. First, we study the dynamics of two merging BECs initially separated by a repulsive dipole barrier. Second, we split a fully formed BEC using the repulsive barriers, and lastly, we explore the effects of releasing atoms from a local high-density region created using an attractive dipole potential. Our experiments begin with ultracold clouds of $^{87}$Rb atoms in the $|F, m_F\rangle = |1, -1\rangle$ hyperfine state constrained in the HIP trap (section 2.5). We choose to work with the $|1, -1\rangle$ hyperfine state because of its long lifetime and large condensed atom numbers in our trap. The trapping frequencies are $\{\omega_{xy}, \omega_z\} = 2\pi \{402, 7\}$ Hz, with the $z$-axis oriented horizontally. Repulsive and attractive barriers for the atoms are created with dipole lasers that are far detuned from the Rb D-lines at 780 and 795 nm (see section 2.3.3). The dipole laser beams are sent horizontally through the center of the magnetic trap, along the radial (tightly confining x-direction). In the vertical direction (y-axis), the laser waist is much larger than the radial extent of the BECs. Dynamics are induced in the BECs by rapidly turning a dipole beam on or off. To enlarge the resulting features, we employ a 2 ms long\(^3\) antitrapped expansion before imaging (see section 2.10 and [68]). During this expansion, the aspect ratio of a BEC\(^4\) formed without the presence of a dipole barrier changes from 57 for the trapped BEC to about 3.

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\(^3\)unless otherwise specified

\(^4\)The aspect ratio is the axial Thomas-Fermi width of the BEC divided by the radial Thomas-Fermi width.
4.3.1 Merging Dynamics

In a first set of experiments, illustrated schematically in figure 4.1, a dipole beam with a wavelength of $\lambda = 660$ nm and waist of $w_z = 27.3\mu$m and $w_y = 32.1\mu$m is used to create a repulsive barrier for the atoms. The beam is turned on before the atoms are evaporatively cooled to form a BEC. After a BEC has formed on either side of the barrier and no surrounding thermal cloud is visible, the dipole beam is rapidly switched off within less than 250 ns and the atoms allowed to fall towards the center of the trap where they merge/collide. We let the dynamics evolve in the HIP trap for a variable evolution time before starting the anti-trapped expansion imaging procedure. The total atom number is about $1 \times 10^6$ atoms, and for a single BEC confined in the HIP trap, this would imply a chemical potential of $\mu = 224\text{nK}$. Figure 4.2 shows an experimental time sequence of the dynamics after the rapid turn-off of the dipole barrier. On the left half of the figure, a relatively weak dipole barrier with a power of $360\mu$W is used. On the right half of the figure, a dipole beam with a power of $3.48$ mW is used, creating a repulsive barrier with a height of $490$ nK for the atoms. As can be seen in figs. 4.2 (k) and (p), the presence of the $3.48$ mW dipole beam leads to two initially spatially separated BECs.

In the case of the weak barrier, only a small suppression in the Thomas-Fermi density profile of the BEC is created near the trap center. Atoms initially on either side of the barrier merge together to form a sharp density peak [fig. 4.2(a), (f)]. Subsequently, this
peak splits into two density peaks which travel towards the edges of the trap along the z-axis [fig. 4.2(b)-(e), (g)-(j)]. An analysis (see below) of the speed at which these density peaks travel confirm that they are sound waves.

The situation for the stronger dipole barrier is noticeably richer. Directly after turning the dipole barrier off, the condensates smoothly expand toward each other [figure 4.2(l), (q)]. This behavior can be described by the well-known dam-breaking problem whereby a sharp density gradient develops into a rarefaction wave (as opposed to a shock wave) when the background density is zero (see, e.g., [52, 66, 67]). Shortly after the BECs have collided near the center of the trap, a pronounced bulge of higher atom density forms in the collision plane [fig. 4.2(m), (r)]. Very pronounced dark notches are observed to form within the high-density bulge as can be seen in fig. 4.2(n), (s). This density bulge then spreads out from the center of the trap [figs. 4.2(m)-(o), (r)-(t)], and more notches are formed to fill the extent of the density bulge with an average spacing of roughly 8 to 11 μm. After about 55 ms, the bulge and the notches have spread over the entire extent of the condensate [fig. 4.2(o), (t)]. The long lifetime, discrete nature, and large amplitude of the notches suggest that they are
Figure 4.3: Numerical simulations provided by Mark Hoefer. Left column, images correspond to experimental time sequence in figure 4.2(k)-(t). The anti-trapped expansion was not simulated; therefore, the vertical scale these images is about 3/57 the vertical scale of figure 4.2(k)-(o), see also [69]. Right column, (f)-(j) simulations showing zoomed-in density slices of the BEC by the plane z = 0 after (f) 5 ms, (g) 6.25 ms, and (h) 7.5 ms.

nonlinear coherent structures rather than simple sound waves.

Numerical simulations\textsuperscript{5} show that a soliton train initially develops and a bulge region is formed where the solitons decay into a large number of vortex rings via a snake instability (see, e.g., [53]), as in figs. 4.3(a)(m) and [69]. Experimentally, vortex rings in BECs have been observed in [64, 70]. They are difficult to detect unambiguously in our experimental images that are integrated along the line of sight. Fine fringes appear adjacent to the bulge region as can be seen, e.g., in figures 4.2(n) and circled in (s). These fine fringes, together

\textsuperscript{5}provided by Mark Hoefer
with the steepness of the wave fronts delimiting the density bulge region, are indicative of DSWs. The merging process finally results in an axial breathing-mode excitation of the BEC.

The qualitative features of the evolution are fairly independent of most experimental parameters. For example, use of two BECs with an initial total atom number of $2.2 \times 10^6$ atoms and a dipole beam with waists of $w_z = 8.5\mu m$ and $w_y = 32.1\mu m$ gives qualitatively the same results as in figure 4.2(k)-(t). However, when the atom number is strongly reduced, we observe a transition in the merging dynamics, both experimentally and numerically, from the generation of a high-density bulge to the generation of a uniform soliton train with no bulge. An experimental time evolution sequence using 22 000 atoms, and a dipole beam power of 150 $\mu W$ is shown in figure 4.4. As in the large atom number case, the two BECs initially separated by the barrier close the density gap smoothly [fig. 4.4(b), (j) and (c), (k)]. In this case, however, no density bulge forms. Instead, dark solitons, appearing as notches in the density profile of the BEC, start emerging soon after the smooth gap closure, and a fully formed soliton train develops at $t = 27$ms.

The transition in the merging dynamics can be understood in the following way. A BEC dark soliton is unstable to long transverse wavelength perturbations leading to vortex formation (see, e.g., [53]). As numerical simulations show in figs. 4.3(f)-(j), the pronounced bulge in figs. 4.2 (m)-(o) coincides with the existence of vortices. By reducing the nonlinearity in the system, we have effectively lengthened the soliton instability wavelength beyond the radial extent of the BEC; thus, the soliton train remains effectively one-dimensional and stable as in figure 4.4 (e)-(h). A one-dimensional analysis reveals that the soliton train can be interpreted as the result of the interaction of two rarefaction waves generated by two dam-breaking problems [55].

The merging dynamics in the low atom case also offers an interesting comparison with matterwave interference patterns observed after releasing two BECs from a trap, as in [3]. In that case, interatomic interactions are largely absent, while they play an important role in our experiments since the BECs merge in the presence of a trapping potential and thus
Figure 4.4: (a)-(h) Experimental anti-trapped expansion images of a BEC with a total atom number of 22,000 merging and forming a soliton train. (i)-(p) Corresponding integrated cross-sections of experimental images on the left.
Figure 4.5: (a) Integrated density of 3D numerical simulations provided by Mark Hoefer for the merging experiment of fig. 4.4. (b) Magnified versions of \( x=0, y=0 \) slices of the 3D numerical simulations. Left: trigonometric interference pattern generated early during the interaction process. Right: soliton train after sufficient merging evolution.

densities are high. A unified viewpoint made by Mark Hoefer [55] gives a hydrodynamic perspective on matterwave interference. In particular, it was shown that the system can be described by elliptic function solutions characterized by an elliptic function parameter. As two clouds merge in trap and the density of the overlapping parts increase, this function parameter changes from 0 to 1, indicating the transition from trigonometric matter wave interference to a soliton train. Figure 4.5 shows numerical simulations performed by Mark Hoefer showing this transition.

### 4.3.2 Splitting a BEC

In a second set of experiments, we investigate the dynamics of splitting a BEC with a repulsive barrier that is suddenly turned on near the center of the trap. Figure 4.6 illustrates
Figure 4.6: Illustrative procedure for splitting a BEC. A BEC is first made (a) before suddenly turning on a repulsive dipole barrier at the center of the condensate (b), inducing dynamics (c) which depend on the strength of the repulsive barrier. The dipole barrier is switched off when anti-trapped expansion imaging begins (d).

Figure 4.7: Dynamics induced by splitting a BEC with a weak repulsive barrier (left two columns) and a stronger repulsive barrier (right two columns).

The steps in this procedure. For very weak barriers that only slightly modify the BEC density, the sudden turn-on merely leads to the propagation of sound waves [71, 72]. Strong barriers, in contrast, lead to DSWs and soliton formation in the wake behind the wave fronts. We first create BECs with $2.2 \times 10^6$ atoms in the HIP trap without the presence of the dipole barrier. Then, a dipole beam with waists of $w_z = 8.5 \mu m$ and $w_y = 32.1 \mu m$ is rapidly turned on and left on for a variable amount of evolution time, after which the antitrapped expansion procedure is started. The dipole barrier is turned off during the 2 ms long anti-trapped expansion just prior to imaging.

The rapid turn-on of a weak dipole beam with a power of 360 $\mu W$ produces two density
peaks which spread out to either side, as shown in figure 4.7(a)-(j). The peak seen in the middle of the BECs in figure 4.7(b)-(e), (g)-(j) is due to merging dynamics that take place during the anti-trapped expansion just prior to imaging (since the barrier is off during the expansion). The two density peaks resulting from the sudden turn on of the repulsive barrier slow slightly as they travel towards the outer edges of the BEC. Therefore, we fit a polynomial of order 2, given by \( a + bx + cx^2 \), to the measurements of position vs. propagation time which fits the data very well as can be seen in figure 4.8(a). We quote the coefficient of the linear term as the central speed of sound. The measured propagation speed of these peaks in the central region of the BECs is plotted in figure 4.8(b) for various powers of the dipole beam. For the lowest powers, the speed is in full agreement with the calculated longitudinal speed of sound, 3.8 mm/s [73, 74, 75]. At these low powers, the density peaks are barely visible in the cloud. For stronger dipole beams, the propagation speed increases above the speed of sound, and for sufficiently strong beams the obtained images change qualitatively due to dispersive shock formation. For laser powers above approximately 0.6 mW, fine fringes appear in front of the propagating peaks. When the dipole beam exceeds a power of roughly 1.2 mW, solitons are formed in the region between the two wave fronts, as shown in figure 4.7(n)-(o), (s)-(t). For an oblate rather than cigar-shaped geometry, related ring-shaped structures have been interpreted in the context of DSWs in [52]. Numerical simulations suggest that this behavior is qualitatively described as follows. Two peaks in the density and outward superfluid velocity are generated by the repulsive dipole beam. These peaks break due to nonlinear steepening [50], causing the generation of two DSWs on the inner and outer edges of each peak. Because of a transverse instability, these quasi-one-dimensional DSWs break up into many vortex rings, leading to interactions. The interaction of DSWs has been studied experimentally in [46, 64] and theoretically in [49]. Deriving an analytic expression for the DSW speeds is complicated by the generation of vortex rings and wave interactions. Empirically, the dependence of pulse propagation speed on the dipole laser power is described reasonably well by a square-root dependence of the form \( v(P) = a + b\sqrt{P} \), as shown by the

6provided by Mark Hoefer
Figure 4.8: (a) Average position of left and right density peak (with respect to center of dipole barrier) vs. propagation time. (b) Speed of wave front propagation through a BEC vs. repulsive barrier laser power. Error bars are taken from fits of distances vs. time plots.

fitted curve in figure 4.8(b). The insets of this figure show two representative images obtained with an evolution time of 12.5 ms and a laser power, respectively, below and above the power at which the wave fronts start breaking into solitons.

Dynamics induced by splitting a BEC and merging BECs can be combined by pulsing on the repulsive dipole barrier for a short period after the formation of a BEC. This is illustrated in figure 4.9(a)-(d). Figure 4.9(e)-(f) shows two representative images at an evolution time of $t = 10$ ms using a weak (left) and a strong (right) dipole barrier. As before, there exists a transition from sound wave propagation to DSW formation and the shedding of dark solitons.

4.3.3 Releasing Dynamics

In a third set of experiments, yet a different prototypical DSW situation comes about when a local high-density peak in a BEC is suddenly released and allowed to spread out into a surrounding background of lower density. We realize this experimentally by replacing the 660 nm dipole laser used in the previous two sections with an 830 nm laser, leading to an attractive dipole potential.

The procedure for this experiment is the same as that illustrated in figure 4.1 but with the repulsive barrier replaced by an attractive dipole beam. In figures 4.10, such an attractive dipole beam with a waist of $w_x = 5\mu$m and $w_z = 41\mu$m and a power of 61 $\mu$W is sent
through the center of the magnetic trap, again in the horizontal radial direction. Evaporative cooling in the presence of the combined optical and magnetic potential leads to a BEC with a localized high-density peak at its center, see figure 4.10 (a). When the dipole beam is suddenly switched off after the formation of the BEC, the central high-density peak spreads out into the surrounding parts of the BEC. Interestingly, the high-density peak does not remain a single peak but quickly splits into two peaks, 4.10 (b), that subsequently travel outwards in opposite directions, figs. 4.10(c)-(e). Such a splitting has also been observed in nonlinear optics [46].

A background density of atoms is required for the single peak to split into two equal parts. Considering a small perturbation on a uniform background with density $\rho_0$, and ignoring the quantum pressure term, one can linearize equation 4.5b to obtain a wave equation of the form

$$\frac{\partial^2 \rho(z,t)}{\partial t^2} = c_0^2 \frac{\partial^2 \rho(z,t)}{\partial z^2}$$

which supports the propagation of two traveling sound waves with speeds $\pm c_0$ [48]. If the initial perturbation is Gaussian in shape, that is $\rho = \rho_0 + A \exp\left(-\frac{2}{\sigma^2}\right)$, a solution is given by

$$\rho(z,t) = \rho_0 + \frac{A}{2} \left[ \exp\left(-\frac{(z-ct)^2}{2\sigma^2}\right) + \exp\left(-\frac{(z+ct)^2}{2\sigma^2}\right) \right],$$

indicating left-traveling and right-traveling Gaussian peaks. For larger perturbations where nonlinear-
Dynamics induced by suddenly turning off a local, attractive dipole potential at the center of the trap. Dipole laser wavelength 830 nm. Left two columns, laser power $61 \mu W$. Right two columns, laser power $183 \mu W$. (a)-(j) sound waves spread away from trap center. (k)-(t) Development of DSWs.

When the initial disturbance is a density dip, the steepening occurs at the trailing edges of the traveling waves. Due to the dispersive nature of the condensate, energy in the shock wave is not dissipated. Instead, fine ripples form ahead of the shock front. In the absence of background atoms, a single peak will remain a single peak that increases in width, just as a Gaussian wavepacket disperses when propagated by the single-particle Schrödinger equation.

For the parameters in figures 4.10 (a)-(d), the measured propagation speed of the two propagating peaks is 4.34 mm/s. When the initial dipole beam power is reduced, the measured speed decreases and approaches the longitudinal speed of sound. When a stronger dipole beam with a power of $183 \mu W$ is used, again two peaks form and spread out (figure 4.10), but now DSWs form, marked by solitons in the inner region and strong ripples in the outer regions of the BEC [figure 4.10 (f)]; see also [69].
Figure 4.11: (a) An initial density bump on a uniform background of atoms splits into two peaks which travel in opposite directions followed by dispersive shock wave formation. (b) A density notch on a uniform background also splits into two oppositely traveling waves. Images provided by Mark Hoefer.

4.4 Chapter Conclusion

Our experiments on merging and splitting of BECs realize several prototypical situations for dispersive shock wave formation and demonstrate the transition from sound wave propagation to dispersive shock dynamics as increasingly stronger perturbations are applied. Several aspects of this behavior, such as the splitting of an initial single peak into two followed by the formation of dispersive shock waves, are very similar to those observed in nonlinear optics [46]. This demonstrates the generality of our results and showcases the usefulness of BECs in the study of nonlinear wave dynamics.

Dispersive shock waves in ultracold Fermi systems are also of interest [76] and have been predicted theoretically in [77]. Utilizing the fermionic $^{40}$K atoms in our system, we perform the same merging experiment described in section 4.3.1 with a degenerate Fermi gas composed of about 300 000 atoms at $T/T_F = 0.27$ and having a chemical potential of $\mu = 810\mu$K. The 660 nm repulsive dipole barrier (with typical waists of $w_x \approx 57\mu$m, $w_z \approx 11\mu$m, and measured power of 3 mW) is turned on prior to the end of sympathetic
cooling of $^{40}\text{K}$ with $^{87}\text{Rb}$ to degeneracy. Figure 4.12(a)-(h) shows a time sequence of the ensuing dynamics. Two density peaks excited by the dipole barrier travel outwards and reach the outer edges of the cloud in figure 4.12(d). They then reverse directions and travel back towards the center of the trap. When the two peaks meet near the center of the trap, they merge smoothly together as in the BEC case. However, no density bulge or DSWs emerge thereafter. Instead, the two density peaks continue to travel outwards again and the gap between the two clouds first created by the dipole barrier reappears [figure 4.12(f)]. The hole re-opening can be understood using a simple picture of non-interacting particles each occupying a harmonic oscillator level of the trap. The reappearance of the hole then is expected to occur at half the axial trapping period and indeed, this is what we observe for tens of cycles or more. The right half of figure 4.12 shows the same experiment but with the
repulsive barrier offset from the center of the trap by \( z = 40 \, \mu m \). The first hole re-opening then occurs at \( z = -40 \, \mu m \) as expected.

The dynamics of figure 4.12 occur in a single species Fermi cloud where the particles are noninteracting. For an interacting cloud, experiments with a two-component \(^6\text{Li}\) mixture at unitarity were performed by John Thomas’ group [78]. The importance of dispersion vs. dissipation for this case is currently a matter of active discussion. Extending our prototypical quantum shock experiments to an interacting Fermi gas presents an intriguing future direction of research for our apparatus as well.
Chapter 5

Two-Component Counterflow Dynamics

5.1 Overview

The observation of dispersive shock waves via the merging and splitting of BECs described in the previous chapter revealed interesting dynamics in which a transition from a regime of sound wave excitations to dispersive shock wave and soliton train formation depended directly on the strength of a repulsive or attractive dipole potential barrier used to manipulate the initial condensate. A natural step forward from this point is to extend the experiments to condensates composed of two distinguishable components. In this chapter, we describe our experiments using two clouds of atoms composed of different hyperfine states of a $^{87}$Rb BEC to investigate the extremely rich dynamics of superfluid-superfluid counterflow.

Nonlinear structures in dilute-gas BECs have been the focus of intense research efforts, deepening our understanding of quantum dynamics and providing intriguing parallels between atomic physics, condensed matter, and optical systems. For superfluids that are confined in a narrow channel, one of the most prominent phenomena of nonlinear behav-

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ior is the existence of solitons in which a tendency to disperse is counterbalanced by the nonlinearities of the system. In single-component BECs, dark and bright solitons, forming local density suppressions and local bumps in the density, respectively, have attracted great interest [19]. In two-component BECs, the dynamics are even richer as a new degree of freedom, the relative flow between the two components, is possible.

Many studies of a superfluid flowing through a normal fluid have been performed (see, e.g. [79]) as a result of the two-fluid model for superfluid helium first formulated by Tisza and Landau in the early 1940s. However, few studies involving the counterflow between two superfluids have been examined in detail. Previous theoretical analysis has demonstrated that spatially uniform, counterflowing superfluids exhibit modulational instability (MI) when the relative speed exceeds a critical value [80]. Modulational instability is characterized by a rapid growth of long wavelength, small amplitude perturbations to a carrier wave into large amplitude modulations. The growth is due to the nonlinearity in the system [81] and we find a critical relative velocity above which a counterflow induced modulational instability sets in. This modulational instability is in turn responsible for the formation of nonlinear structures such as dark-bright solitons and the novel oscillating dark-dark soliton. Our experiments and analysis reveal that by carefully tuning the relative speed slightly above the critical value, we can enhance large amplitude density modulations at the overlap interface between two nonlinearly coupled BEC components while mitigating the effects of the instability in the slowly varying background regions. A dark-bright soliton train then results.

In two previous experiments, individual dark-bright solitons were engineered in two stationary components using a wave function engineering technique [82, 83]. In our experiment we find that trains of dark-bright solitons can occur quite naturally in superfluid-superfluid counterflow. This novel method of generating dark-bright solitons turns out to be robust and repeatable. In single-component, attractive BECs the formation of a bright soliton train from an initial density jump has been predicted [84]. However, both condensate collapse and the effects of MI in the density background must be avoided, placing restrictions on the confinement geometry and diluteness of the single component condensate. In contrast,
the properties of counterflow in miscible, two-component BECs, as we show, enable the
observation of trains consisting of ten or more dark-bright solitons in BECs with a large
number of atoms. We also note that modulated soliton trains have been studied extensively
in single-component, modulationally stable repulsive BECs where supersonic flow supports
the generation of dispersive shock waves [53, 51, 52, 85]. The dark-bright soliton train we
study in this work occurs when one or more of the system’s sound speeds become complex
so that the standard definition of supersonic flow does not apply.

5.2 Two-Component Gross-Pitaevskii Equation

The dynamics of two-component BECs can be described with the mean-field coupled GP
equations

\[ i\hbar \frac{\partial \Psi_1(r,t)}{\partial t} = \left( -\frac{\hbar^2}{2m_1} \nabla^2 + V_1 + \frac{4\pi\hbar^2 a_{11}}{m_1} |\Psi_1|^2 + \frac{2\pi\hbar^2 a_{12}}{m_{12}} |\Psi_2|^2 - \mu_1 \right) \Psi_1 \]

(5.1a)

\[ i\hbar \frac{\partial \Psi_2(r,t)}{\partial t} = \left( -\frac{\hbar^2}{2m_2} \nabla^2 + V_2 + \frac{4\pi\hbar^2 a_{22}}{m_2} |\Psi_2|^2 + \frac{2\pi\hbar^2 a_{12}}{m_{12}} |\Psi_1|^2 - \mu_2 \right) \Psi_2 \]

(5.1b)

where \( \mu_1 \) and \( \mu_2 \) are the chemical potentials of the two components and the atomic masses
\( m_1 = m_2 \) in our case as we use two hyperfine spin states of the same atomic species. The
reduced mass is given by \( m_{12} = m_1 m_2 / (m_1 + m_2) \). The intra-component s-wave scattering
lengths are denoted by \( a_{11} \) and \( a_{22} \) and the inter-component scattering length is \( a_{12} \), all of
which are positive for \(^{87}\)Rb. Mean-field theory predicts that a mixture is miscible if [86, 87, 88]

\[ a_{12} < \sqrt{a_{11} a_{22}}. \]

(5.2)

We consider uniform counterflow between the two components in which the second com-
ponent moves with a velocity \( \mathbf{v} \) relative to the first. Thus, we can write \( \Psi_1 = \sqrt{\rho_1} e^{i\mathbf{k} \cdot \mathbf{r}} \) and
\( \Psi_2 = \sqrt{\rho_2} e^{i\mathbf{k} \cdot \mathbf{r}} \) as solutions to the coupled GP equations. Here, \( \mathbf{k} = m\mathbf{v}/\hbar \), and \( \rho_1 \) and \( \rho_2 \)
are the number densities of components 1 and 2, respectively.

The coupled GP equations can also be written in the hydrodynamic form as was done for
the single component GP equation in section 4.2. Since our trapping geometry is a highly
elongated cigar shape with transverse trapping frequencies $\omega_x \approx \omega_y$ much greater than the
axial trapping frequency $\omega_z$, we consider the dimensionless coupled GP equations in 1+1
dimensions given by

$$
\begin{align}
&i \frac{\partial \psi_1}{\partial t} = -\frac{1}{2} \frac{\partial^2 \psi_1}{\partial z^2} + g_{11} |\psi_1|^2 \psi_1 + g_{12} |\psi_2|^2 \psi_1 \\
&i \frac{\partial \psi_2}{\partial t} = -\frac{1}{2} \frac{\partial^2 \psi_2}{\partial z^2} + g_{22} |\psi_1|^2 \psi_2 + g_{12} |\psi_2|^2 \psi_2
\end{align}
$$

(5.3a)

(5.3b)

with distance in units of the transverse harmonic oscillator length $\sqrt{\hbar/(m\omega_x)}$, time
in units of $1/\omega_x$, $g_{ij} = 4\pi a_{ij}$, and the axial confinement neglected. Plugging $\psi_j(z,t) = \sqrt{\rho_j(z,t)} e^{i\phi_j(z,t)}$ into the above equations and separating real and imaginary parts, one can
write the nondimensional hydrodynamic formulation of the coupled GP equations as

$$
\begin{align}
&\frac{\partial \rho_j}{\partial t} + \frac{\partial}{\partial z} (\rho_j u_j) = 0 \\
&\frac{\partial u_j}{\partial t} + \frac{\partial}{\partial z} \left( \frac{1}{2} u_j^2 + \rho_j + \sigma_j \rho_{3-j} \right) = \frac{1}{4} \frac{\partial}{\partial z} \left[ \frac{1}{\rho_j} \frac{\partial^2 \rho_j}{\partial z^2} - \frac{1}{2 \rho_j^2} \left( \frac{\partial \rho_j}{\partial z} \right)^2 \right]
\end{align}
$$

(5.4a)

(5.4b)

where the index $j = 1, 2$ refer to the two components, $\sigma_j = a_{12}/a_{jj}$, $u_1 = -v/2$ and
$u_2 = v/2$. The 3D densities can be approximated by the harmonic oscillator ground state
via $\rho_j(z,t) \times \exp \left( -x^2 - \frac{\omega_y}{\omega_x} y^2 \right)/(2\pi a_{jj} a_0^2)$, with $a_0$ being the Bohr radius.

Following [80], in which small perturbations proportional to $e^{i(\kappa z - \omega t)}$ are applied to the
plane wave solutions and a Bogoliubov-deGennes type analysis is performed, it can be demonstrat
that counterflow leads to complex $\omega$. In the case when $\text{Im} \omega(\kappa) > 0$, modulational
instability appears in the system and occurs for $v$ larger than a critical velocity $v_{cr}$ with
a maximum growth rate $g_{\text{max}} = \text{Im} \omega_{\text{max}}$ and associated wave number $\kappa_{\text{max}}$. Repeating the
calculation in [80], we find for the homogeneous case $^2 v_{cr}$ to be bounded by

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"theoretical support is provided by Mark Hoefer"
Figure 5.1: Illustrative summary of the main experimental steps used to induce superfluid-superfluid counterflow. (a) A single component BEC is trapped in an optical dipole beam and an ARP is applied, resulting in (b), a perfectly overlapped mixture of atoms in two different hyperfine states. (c) Counterflow between the two components is induced with an external magnetic gradient. (d) The resulting dynamics of both components are imaged.

\[ \sqrt{\rho_1(1-\sigma_1\sigma_2)} \leq v_{cr} \leq 2\sqrt{\rho_1(1-\sqrt{\sigma_1\sigma_2})}, \quad \rho_1 \geq \rho_2. \quad (5.5) \]

The lower bound is valid for small \( \rho_2/\rho_1 \) and the upper bound is applicable for \( \rho_2 \sim \rho_1 \). The modulational instability can be exploited to create intriguing solitonic structures, as shown in our experiments below.

5.3 Superfluid-Superfluid Counterflow Experiment

Preparation of the two-component BEC begins with the loading of a fully formed (single component) \(^{87}\text{Rb}\) BEC from the magnetic trap to the single beam 1064 nm optical dipole trap (described in detail in section 2.9). The experimental procedure for our superfluid-superfluid counterflow studies contains three main steps. After loading into the dipole trap, a variable amount of atoms from a particular initial hyperfine state is transferred to a second hyperfine state, resulting in a two component BEC. Then counterflow between the two components is induced by applying an external magnetic gradient. The last step is to image the atoms after some in-trap evolution time. Figure 5.1 summarizes the main steps of the experiments described in this chapter.

Preparing a Two-Component Condensate– The loading of atoms into our dipole trap is discussed in section 2.9. The measured trapping frequencies for the experiments performed in this section are \( (\omega_x, \omega_y, \omega_z) = 2\pi \times (174, 120, 1.2) \) Hz unless otherwise stated. A 1 ms long microwave adiabatic rapid passage (ARP) transfers a variable amount of atoms from the
initial state to a second state within the ground state hyperfine manifold. This second state is chosen such that its Zeeman energy shift is opposite from that of the initial state. If our initial BEC is composed of atoms in the $|2, 2\rangle$ ($|1, -1\rangle$) state, the second state chosen is the $|1, 1\rangle$ ($|2, -2\rangle$) state. We find no qualitative difference in the behavior of the dynamics depending on which pair of states are used. This is not surprising as the s-wave scattering lengths for the $|1, \pm 1\rangle$ and $|2, \pm 2\rangle$ states, as well as inter-component scattering lengths of the two pairs are the same. The initial atom number prior to the ARP is about 450 000 atoms all in the $|2, 2\rangle$ state and about $8 \times 10^5$ atoms if starting with the $|1, -1\rangle$ state. A 1.1 Gauss magnetic bias field oriented along the vertical direction lifts the degeneracies between the hyperfine states. The precise location of the transfer resonance is mapped out by looking at the number of atoms transferred to the second state (or equivalently the atoms left in the initial state) while varying the microwave frequency of short, fixed frequency microwave pulses. A typical resonance curve is shown in figure 5.2 where a fit to the data locates the position of the peak. Once the resonance is located, the relative number of atoms in the two components can be controlled by adjusting the speed or the intensity of the microwave sweep across the resonance that produces the mixture. After the ARP, a perfectly overlapped mixture of the two states is produced. The scattering lengths for the two states used in our experiment are estimated to be $a_{11} = 100.40$ a.u. and $a_{22} \approx a_{12} = 98.98$ a.u. [89], therefore according to equation 5.2, our system is predicted to be weakly miscible. In contrast, previous studies of two-component $^{87}$Rb BECs concentrated mostly on the states which are immiscible [90, 91], with the notable exception of [92].

Figure 5.2: A typical resonance curves for locating the ARP microwave resonance from the $|1, -1\rangle$ to the $|2, -2\rangle$, $|2, -1\rangle$ and $|2, 0\rangle$ states.
Figure 5.3: Mixture miscibility. A 50/50 mixture of atoms in the $|1,1\rangle$ and $|2,2\rangle$ states form a miscible mixture in the absence of an external magnetic gradient, (a)-(c). (d)-(f) The two states separate completely along the z-direction when an axial gradient leading to a trap shift of about 60 $\mu$m is applied. In-trap evolution times are indicated in each image.

When the overlapped mixture is allowed to evolve in trap undisturbed by any external gradients, we observe no phase separation over the experimental time scale of several seconds, in agreement with the predicted miscibility of the two components and is demonstrated in figure 5.3(a)-(c). The upper cloud of each image in this figure shows the atoms in the $|2,2\rangle$ state at a time 7 ms after a sudden turn-off of the optical trap, while the lower cloud, taken during the same experimental run, shows the atoms in the $|1,1\rangle$ state after 8 ms of expansion (see below for imaging procedure). During their in-trap evolution, the $|1,1\rangle$ and $|2,2\rangle$ clouds are perfectly overlapped in the vertical direction. The dominant effect of the time evolution in figure 5.3(a)-(c) is a slow decay of the atom number over time. For single-component BECs, we have measured an exponential BEC lifetime of over 50 s for the $|1,1\rangle$ state and 14 s for the $|2,2\rangle$ state in our dipole trap. Motion induced by changes of mean-field pressure during the decay may be responsible for a small scale roughness of both components which becomes visible after several seconds [fig. 5.3(c)].

Inducing Counterflow–Since the two components of our BEC are composed of maximally stretched states with opposite magnetic moments, the two components experience forces which pull them in opposite directions in the presence of an externally applied magnetic
gradient. This is equivalent to applying a differential shift between the harmonic potentials for the two components along the axial direction of the trap. Counterflow is induced as the two components accelerate in opposite directions. Our applied magnetic gradients are created using a pair of small bias coils (figure 2.16) oriented along the axial direction of the dipole trap and the gradient is such that the low-field seeking $|2, 2\rangle$ ($|1, -1\rangle$) state is pulled to the right and the high-field seeking $|1, 1\rangle$ ($|2, -2\rangle$) state is pulled towards the left. The gradient can either be jumped on abruptly or be ramped on. While both methods lead to qualitatively the same dynamics, we find that ramps lead to more reproducible dynamics. In figure 5.3(d)-(f), a gradient leading to a differential trap shift of $\sim 60 \mu m$ is applied to an initially perfectly overlapped 50/50 mixture. The gradient is left on for the duration of the in-trap evolution. At $t = 100 ms$, counterflow is visible as the two components separate and after an evolution time of 9 s, the two clouds are nearly completely demixed. Note that this demixing occurs even for relative trap shifts that are small compared to the overall extent of the cloud ($60 \mu m$ vs. $500 \mu m$), indicating the importance of inter- and intra-species interactions for the demixing.

**Imaging Procedure**—In order to produce data which may be analyzed more easily, images of both components of the BEC are captured in a single experimental run. Doing so also reduces data taking time by half and, more importantly, allows us to correlate structures in the two clouds that would otherwise be subject to random spatial shot-to-shot noise. In all the experimental images shown in this chapter, we begin the imaging procedure by abruptly turning off the magnetic gradient as well as the optical dipole trap, and allow the BEC to expand freely. Because of the elongated shape of the dipole trap, the expansion occurs almost exclusively in the radial direction. During the expansion, the density rapidly drops and mean field driven dynamics ceases, so that the expanded images are good representations of the in-trap densities. After 7 ms of free expansion, we selectively image the atoms in the $|2, 2\rangle$ (or $|2, -2\rangle$) state by resonant absorption imaging on the $F = 2$ to $F' = 3$ transition. We then apply a 1 ms long microwave sweep that swaps the hyperfine states of the atoms between the $|1, 1\rangle$ and $|2, 2\rangle$ (or the $|1, -1\rangle$ and $|2, -2\rangle$) states, and image again using the
same imaging transition as before. During the 1 ms microwave sweep the atoms continue to fall under the influence of gravity, so that the second image appears below the first on the CCD camera. Due to the predominantly radial character of the expansion and the low densities, the difference in expansion times for the two components (7 ms and 8 ms, resp.) has no noticeable effect in the axial direction and only leads to a slightly larger radial expansion factor for the second image, in addition to the desired effect of offsetting the images vertically on the camera.

### 5.4 Two-Component Merging Dynamics

Extending upon the BEC collision experiments of chapter 4, we first inspect the merging of a two component BEC. We begin with the situation in figure 5.4(a) [also 5.3(f)] where the two components are completely separated from each other along the axial direction.
When the gradient is suddenly jumped off, the two components interpenetrate, first forming a smooth and extended overlapping region. In the ensuing dynamics, a curious structure resembling a bright soliton sitting directly next to a dark soliton in the $|2, 2\rangle$ state (and vice versa for the $|1, 1\rangle$ state) is observed, as can be seen in figure 5.4(b) and (d). This nonlinear structure turns out to be a type of vector soliton known as the oscillating dark-dark (DD) soliton [93, 94]. The formation of this DD soliton is reminiscent of dark soliton formation in the single-component BEC collision experiments of chapter 4. DD solitons are also theoretically predicted to develop when a repulsive beam is swept through a two-component miscible BEC with an appropriate speed [94]. However, our experiment is the first, and so far only, to demonstrate these features in the lab. It is revealed through further experiments and theoretical work (see following sections and fig. 5.10) that the density profile of the DD soliton is not static. Over time, energy between the two components can be exchanged such that the dark-next-to-bright density profile of one component becomes a bright-next-to-dark density profile (and vice versa for the second component). Theoretically, the DD soliton can be understood as a “rotated” (in component space) version of a dark-bright soliton [95], see section 5.5.2. To investigate how the DD soliton is formed, we look in detail at the dynamics during the mixing and demixing of the two components.

5.5 Counterflow Dynamics

The following section is divided into three subsections in which we explore the surprisingly rich dynamics found for three regimes of relative counterflow speeds: i) fast, ii) moderate, and iii) slow. In each of these cases, a counterflow-induced modulational instability is responsible for the observed dynamics.

5.5.1 Modulational Instability and Fast Counterflow

Modulational instability (MI), in which small perturbations to a carrier wave, reinforced by nonlinearity, experience rapid growth [81], plays a key role in our superfluid-superfluid
counterflow dynamics. In many nonlinear systems, MI leads to the breakup of periodic wave trains, as in sufficiently deep water [96], as well as the formation of localized structures in optics [97, 98] and BECs [99].

Starting with BECs composed typically of $8 \times 10^5$ $^{87}$Rb atoms in the $|1, -1\rangle$ state and transferring $\sim 50\%$ of the atoms to the $|2, -2\rangle$ state via a 1 ms long microwave sweep, a perfectly overlapped two-component mixture is created as in figure 5.3(a). The measured trapping frequencies for this set of experiments are $(\omega_x, \omega_y, \omega_z) = 2\pi(178, 140, 1.5)$ Hz. A magnetic gradient of 10.4 mG/cm corresponding to a relative trap shift of 176 $\mu$m is applied to the mixture to induce a strong counterflow. Figure 5.5 showcases experimental data of the formation of a very dense counterflow-induced MI pattern. After smooth evolution for nearly 60 ms, a gradual pattern of small density modulations start to form at $t=70$ ms [5.5(b),(i)]. We first observe pattern formation in noncentral regions where the two conden-
sates have differing densities. This is due to the dependence of the critical velocities for counterflow-induced MI on the two-component density ratio being largest when the densities are equal [100, 80]. After ~95 ms, a very dense and regular MI pattern fully develops, filling the entire BEC [fig. 5.5(d),(k)]. The modulations in the two components are offset in the axial direction in a staggered way such that one component fills the depressions in the other and the overall density profile remains fairly smooth, as can be seen in the integrated cross sections of figure 5.5(k). Under the continued influence of the axial gradient, the regular pattern of figure 5.5(d) quickly becomes uneven and irregular [fig. 5.5(e)-(g),(l)-(n)]. Alternatively, if the gradient is switched off after the MI pattern has fully developed, we frequently observe the formation of black dots such as those marked by the arrows in figure 5.6(a), which may indicate the generation of vorticity. We note recent theoretical work suggesting that counterflow-induced MI may be used to generate quantum turbulence [101].

MI theory agrees quantitatively with the experimentally observed patterns as we now explain (fig. 5.7). For a uniform counterflow, the onset of MI corresponds to a complex sound speed and exhibits a preferred wave number \( \kappa_{\text{max}} \) corresponding to the maximum

Figure 5.6: Evolution of regular MI pattern in the absence of an applied gradient. White arrows indicate possible vortices. Dashed boxes indicate DB soliton mergers.
growth rate $g_{\text{max}}$, both depending on the counterflow speed. Unfortunately, our imaging procedure does not allow us to determine the counterflow speeds experimentally. However, following an analysis provided by Mark Hoefer (NCSU), one can take two independent theoretical approaches, described below, to extract the onset velocities from our experimental data. The fact that these two independent approaches lead to consistent results gives quantitative credence to the theory. First, we use the analytical theory in references [9] and [80] to calculate the counterflow speed $v_{\text{fit}}$ whose corresponding $\kappa_{\text{max}}$ equals the experimentally observed pattern periodicity at the trap center where the densities are assumed to be equal (the solid, black curve in fig. 5.7). In a second, independent approach, we assume spatially uniform counterflow whereby the applied gradient leads to unimpeded acceleration of each component (calculated from the atomic magnetic moment and the magnitude of the applied gradient). Using this simple model, experimentally determined onset times for MI are converted to relative speeds at the onset of the MI pattern (the dashed blue curve in fig. 5.7). The dashed-dotted red curve in fig. 5.7 uses the same uniform counterflow model but shifts the measured MI onset time by $1/g_{\text{max}}$. Subtracting this time accounts for the development of the instability and leads to a better approximation of the true relative speed that sets the pattern periodicity. The resulting curve interpolates the two models. The lowest dotted curve is the predicted critical speed in the condensate center ($v_{\text{cr}} = 0.16 \text{ mm/s}$) demonstrating fast counterflow. For comparison, the central speed of sound in a single-component BEC with the same total atom number is $\sim 1.6 \text{ mm/s}$. Despite the approximations made, the curves exhibit agreement for small-to-moderate gradients, suggesting that the observed dynamics are theoretically described by counterflow-induced MI. Discrepancies at large gradients are likely due to the large accelerations involved and spatial nonuniformity.

5.5.2 Oscillating Dark-Dark Solitons

In the following we explore moderate counterflow speeds between the two superfluid components and find that modulational instability-induced regular density modulations, formed throughout the BEC, lead to the emergence of a large number of beating dark-dark solitons.
These solitons—which exhibit periodic energy exchange between the two condensate components [93]—are a generalization of static dark-dark solitons [102]. The dark-dark solitons we observe here are unique and distinct from the dark-bright solitons that have been observed previously in BECs [82, 83] and in the next section of this chapter, which are distinguished by their far-field conditions and dynamics. A dark-bright soliton consists of a dark notch in one component, filled by a localized density bump of the second component, as shown in figure 5.8(b), (c). In contrast, the beating dark-dark soliton asymptotes to nonzero densities in both components and dynamically changes its shape, with each component possessing a density bump adjacent to a notch which alternate their relative positions in time (see fig. 5.8(d)-(l), also fig. 5.10).

In figure 5.8, a gradient of 1.4mG/cm leading to a relative trap shift of about 23µm is applied to a 50/50 mixture of atoms. For this gradient, $\kappa_{max}$ is reduced relative to figure 5.5, enabling better experimental observation of individual features. For many milliseconds, the clouds flow through each other smoothly and there is little visible change. Then, as in the large applied gradient case, MI sets in across the BEC, leading to a regular array of dark-dark solitons [figs. 5.8(e)-(l)]. In accordance with theory and numerics, the DD solitons exhibit...
Figure 5.8: Onset of MI pattern containing oscillating dark-dark solitons for moderate counterflow speeds above the critical velocity. (b), (f), (j) Zoomed in region of dashed box in (a), (e), (i), respectively. (c), (h) and (l) Integrated cross sections of (b), (f) and (j), respectively. (g) and (k) are integrated cross sections of (e) and (k). Black curves correspond to the $|2, -2\rangle$ state, red curves correspond to the $|1, -1\rangle$ state, and blue curves indicate the sum of the two states.
a dynamic beating, as seen by comparing the integrated cross sections of figures 5.8(h),(l), noting the order of the notch and bump feature in each component. While our destructive imaging technique does not allow us to determine the exact beat frequency, 3D numerics\(^3\) indicate a time scale of 15 ms per period.

The dynamics are well reproduced by 3D numerical simulations\(^4\) of the vector, mean-field GP equation with initial conditions and parameters corresponding to the experiment in figure 5.8. As with experiment, a smooth, accelerating counterflow develops due to the axial field gradient. Dark-bright solitons form at the edges of the condensates until the rapid growth of large-scale modulations is observed [figs. 5.9(a),(c)]. For the moderate gradients in figures 5.9(a), (c), and (e), these modulations rapidly develop into a number of localized, essentially one-dimensional (1D) beating dark-dark solitons with an initial approximate spacing \(2\pi\kappa_{\text{max}}\). Continued evolution results in interactions and eventual solitary wave transverse breakup at approximately \(t = 600\) ms.

The numerics also reveal that the axial modulations rapidly undergo decay due to transverse modulations, which leads to the formation of columnar 2D vortex lines [figures 5.9(b) and (d)], exhibiting a \(2\pi\) phase winding around their core [fig. 5.9(f)], and a uniform structure along the direction of view. The numerics also show vortex lines oriented along the orthogonal, horizontal radial axis. In analogy to the scalar case \([103, 104]\), we interpret this behavior as a transverse instability that depends on the relative speeds of the two components, their densities, and the transverse confinement strength.

The beating DD solitons can be understood through the following simplified model: Assuming that all scattering lengths are equal to \(a_{22}\), the mean-field equation is the repulsive, vector NLS equation. Its most general known soliton solution is the six-parameter dark-dark soliton \([93]\) (e.g., two background densities \(n_{1,2}\), two background flow speeds \(c_{1,2}\), soliton speed \(v\), and beating frequency \(\omega\)) of which the well-studied five-parameter static dark-dark soliton \([102]\) is a special case. Even though analytical expressions for these solitons were derived \([93]\), their form is quite complicated and basic properties such as the beating

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\(^3\)Provided by Mark Hoefer (NCSU).
\(^4\)Provided by Mark Hoefer (NCSU).
frequency as a function of soliton parameters are unknown.

An example of a beating dark-dark soliton can be constructed by leveraging the SU(2) invariance of the vector NLS equation in the case where all scattering lengths ($a_{11}$, $a_{22}$, and $a_{12}$) are equal. Applying a rotation matrix to the two components of a four-parameter dark-bright soliton [102], we obtain a five-parameter beating dark-dark soliton where the background flow speeds are equal to $c$. Its evolution over half a beating period is shown in figure 5.10 [compare with figs. 5.8(h), (l) and fig. 5.9(e)]. The beating angular frequency $\omega = \frac{m}{2\hbar}(cv)^2 \sec^2(\phi/2)$ satisfies

$$m(cv)^2/(2\hbar) < \omega < \pi\hbar a_{22}(n_1 + n_2)/m.$$  \hspace{1cm} (5.6)

The soliton half-width is $l = h/\sqrt{2m\omega\hbar - m^2(c-v)^2}$, where $\phi$ is the soliton phase jump and $m$ is the particle mass. As $\omega$ approaches the lower (upper) bound in the above equation, the beating soliton degenerates to a plane wave (static dark-dark soliton). While the
scattering lengths in our physical system are only approximately equal, the beating soliton constructed above strongly resembles features observed in the experiment and numerical simulations. The predicted minimum oscillation period of 5 ms for our experimental parameters is consistent with the numerically observed periods of $\sim 15$ ms.

### 5.5.3 Dark-Bright Soliton Train via Counterflow

We now investigate the dynamics induced by small gradients and show how they can be exploited to create dark-bright (DB) soliton trains. In figure 5.11 an initially overlapped mixture of 30% of the atoms in the $|2, 2\rangle$ state and 70% in the $|1, 1\rangle$ state is used. A small magnetic gradient in the axial direction is linearly ramped on over a time scale of 1 s, leading to a calculated trap separation for the two species of only about 3 $\mu$m. After the end of this ramp, the gradient is held constant. The evolution times given in figure 5.11 do not begin until after the 1 s gradient ramp is completed. Therefore, the two clouds in fig. 5.11(a) are already beginning to separate along the axial direction. In the subsequent evolution, individual stripes break off one after another from the left edge of the $|2, 2\rangle$ component,
Figure 5.11: Formation of a dark-bright soliton train during superfluid-superfluid counterflow. (a)-(f) Experimental time sequence. (g)-(l) Integrated cross sections of experimental image directly to the left. Red curves correspond to the $|2, 2\rangle$ state while the black corresponds to the $|1, 1\rangle$ state.

and perfectly aligned dark notches appear in the $|1, 1\rangle$ component. The predominantly uniform widths of the observed stripes and notches, their long lifetime of several seconds in the absence of a magnetic gradient, as well as their dynamics resembling individual stable entities (see below) are strong experimental indications that the observed features are indeed dark-bright solitons.

The DB soliton train formed via superfluid-superfluid counterflow demonstrates the reproducibility and robustness of these solitons. Even though each image is a separate experimental run, they can all still be assembled to form a consistent sequence (i.e. the number of solitons which form are not overly sensitive to field drifts and other initial conditions).

The observed soliton formation is reproduced by three-dimensional (3D) numerical simulations\(^5\) of the two-component Gross-Pitaevskii (GP) equations. Parameters used for the

\(^5\)provided by Mark Hoefer
Figure 5.12: 3D numerical simulations provided by Mark Hoefer (NCSU) modeling the experiments of fig. 5.11. (g)-(i) are integrated density distributions of (a), (b), and (e) respectively with the red curve corresponding to the top cloud and the red curve corresponding to the bottom cloud. (j) is a zoomed in view of (i) at the dark-bright solitons. (k) shows the phases of the two components.

GP equations are the experimental values. These values lead to dynamics that closely match the experiment, as shown in figure 5.12 with a moderate time delay of a little more than one second, as can be seen by comparing figure 5.11(a)-(f) with figure 5.12(a)-(f). Our numerical calculations suggest that the time delay may be due to uncertainties in the estimated magnetic field gradient-induced trap shifts. The experimentally invoked free expansion directly before imaging the condensate was not performed in the numerical simulations.

Numerical results for the quantum mechanical phases of the two wave functions describing the components are shown in figure 5.12(k). The nearly linear phase behavior on the right (at $x \geq 50 \mu m$) indicates a smooth counterflow of the two components. In the soliton region, the phase jumps across the dark solitons as well as the phase gradients in the bright component vary slightly, so that the dark-bright solitons are moving relative to one another, which eventually leads to dark-bright soliton interactions.

The unequal atom numbers in each component leads to a pronounced density jump near the edge of the $|2, 2\rangle$ state with the application of the external gradient. Since $v_{cr}$ is dependent on the density ratio of the two components, being largest when the density ratio is one, the instability first sets in near the density jump. Thus, the global onset of the MI throughout the entire cloud is suppressed while locally, near the density edge, the instability
Figure 5.13: (a)-(i) Experimental images of the in-trap oscillation of a single DB soliton. For this set of data, the oscillation frequency was measured to be about 0.27 Hz while the axial trapping frequency is $2\pi \times 1.2$ Hz. (j) In-trap oscillation of DB solitons. Red triangles: $N_B \approx 680$, $N_D \approx 27,000$ atoms, the fitted curve gives an oscillation frequency of 0.39 Hz; blue dots: $N_B \approx 9000$, $N_D \approx 650,000$ atoms, fit gives 0.27 Hz. (k) red data points are for $N_D \approx 30,000$ atoms, green data points are for $N_D \approx 200,000$ atoms, and blue data points are for $N_D \approx 430,000$ atoms.

is enhanced. The instability manifests itself as modulations at the density jump which then grow to form a train of dark-bright solitons.

By reducing the initial number of atoms in the component forming the bright soliton (i.e. the $|2, 2\rangle$ or $|2, -2\rangle$ state), we can also reliably produce individual dark-bright solitons and observe their oscillation in trap, similar to the dynamics observed in [83]. In figure 5.13, a 1 s long ramp-on of the axial gradient is applied to the mixture. Immediately after the soliton is fully formed, we jump off the gradient and watch its in-trap oscillations. We find, as in [83], the oscillation frequency to be much slower than the dipole axial trapping frequency of $2\pi \times 1.5$ Hz. We also find a dependence of the in-trap oscillation frequency on the number of atoms in the bright component. By varying the microwave sweep used to create the two-component mixture, the number of atoms transferred to the bright component of the DB soliton ranges from about 680 to 12 000 atoms. Figure 5.13(k) shows our results where, in general, solitons with larger bright components are observed to oscillate more slowly.
Two DB solitons can also be generated, once again, by tuning the microwave power for the ARP. In figure 5.14, two such solitons are made and their in-trap evolution observed. Although destructive imaging is used to capture these dynamics and do not allow us to extract any phase information about the solitons, it is clear that the solitons behave as individual entities by emerging from a collision without visible changes in all other regards.

In figure 5.15, we demonstrate the feasibility of studying interactions of dark-bright solitons with Gaussian potential barriers. Here, a repulsive dipole barrier (derived from our 660 nm dipole laser, see section 2.3.3) affecting both BEC components equally, either transmits [fig. 5.15(a)] or reflects [fig. 5.15(b)] dark-bright solitons initially on the left side of the barrier. The “reflectivity” depends on the barrier height; compare fig. 5.15(a)-(d) to fig. 5.15(e)-(h). The situation becomes far more interesting if a species selective dipole barrier could be used. For example, the theoretical analysis in [105] predicts that a repulsive barrier for one component of the BEC counterintuitively amounts to an attractive barrier for the second component due to the repulsive intercomponent interactions.
Figure 5.15: Experimental images of DB solitons (a)-(d) penetrating a weak (barrier height \( \sim 0.56\mu_{BEC} \)) repulsive dipole potential, and (e)-(h) reflected from a strong (barrier height \( \sim 1.11\mu_{BEC} \)) repulsive dipole potential. Evolution times are (a) 250 ms, (b) 500 ms, (c) 750 ms, (d) 1000 ms, (e) 300 ms, (f) 600 ms, (g) 700 ms, and (h) 900 ms. The chemical potential of the BEC is \( \approx 36 \text{ nK} \).

5.6 Chapter Conclusion

Surprisingly rich dynamics have been observed in the relatively simple setting of superfluid-superfluid counterflow, providing a robust method for creating individual and trains of dark-bright solitons. Our counterflow method also produced the first experimental observation of the beating dark-dark soliton in a BEC. The DD soliton naturally arise from a fast counterflow-induced modulational instability and can emerge during the mixing of two superfluids as well.
Chapter 6

Summary and Outlook

In this thesis, a number of prototypical quantum hydrodynamic situations for the generation and study of dispersive shock waves and nonlinear defect structures have been realized.

Intriguingly rich dynamics can often be found in surprisingly simple settings such as the merging of two trapped single-component BECs. A regular dark soliton train forms and the system can be described using elliptic function solutions which transition from trigonometric interference patterns, formed in the initial low density overlapping region of the colliding condensates, to a soliton train. When the total number of atoms in two colliding BECs is large (about 1 million atoms), and the two BECs are initially completely separated, a striking region of high atom density forms where the BECs first meet and sharp density edges (shock fronts) propagate outwards along the collision direction, increasing the width of the high density bulge. As the dynamics evolve, dark (grey) solitons fill the bulge and can decay into vortices via the “snake” instability [53, 70].

Exceptionally rich dynamics are found when the single component collision experiments are extended to two-component BECs. Here, a study of superfluid-superfluid counterflow is performed in which we find a critical counterflow velocity above which modulational instability sets in. Modulational instability plays an important role in the dynamics we observe, setting the length scale of the resulting defect structures and allowing for the realization of dark-bright solitons as well as novel oscillating dark-dark solitons. The dark-dark solitons
have a non-static density structure and have been studied theoretically. However, our exper-
iment is, to the best of our knowledge, the first to realize such a structure in the lab. The
dark-bright solitons in our counterflow system form as a result of the larger intercomponent
density differences near the edges of the BEC. The critical velocity, which is lower for larger
density differences, allows modulational instability to set in first near the BEC edge, while
the nearly equal intercomponent densities elsewhere in the BEC suppress the onset of the
instability. The resulting dark-bright solitons, produced either individually or in a train, are
extremely robust and long-lived (on the order of several seconds). Some first studies using
the dark-bright solitons generated using our counterflow method include single dark-bright
soliton oscillations in a harmonic-trap, collisions between two dark-bright solitons, and in-
teractions of dark-bright solitons with a repulsive external potential barrier which is present
in both components.

The experiments presented in this thesis open the door for a variety of further studies.
Future directions for expanding upon our investigations include the construction of a phase-
contrast imaging scheme to study the dynamics of two interacting dark-bright solitons in
detail. A phase-contrast imaging scheme would also allow us to study the beating frequency
of the dark-dark solitons, which is numerically calculated to be on the order of \(\sim 15 \text{ ms}\).
Another advantage of phase-contrast, or any in-situ imaging technique, is to decrease data
taking time while also eliminating shot-to-shot uncertainties in position, atom numbers, etc.

In addition to our counterflow-induced modulational instability studies, the two-component
experiment can be modified to investigate a variety of other hydrodynamic instabilities in a
quantum system, such as the Rayleigh-Taylor (RT) and Kelvin-Helmholtz (KH) instabilities.
The RT instability arises at the interface between two fluids of differing densities and grows
in time when a perturbation is applied [106, 107]. When there is a difference in the velocity
of the two fluids parallel to the interface, KH instabilities can occur [108]. To study these
instabilities in our experiment, we can separate the two-components of the BEC in the ver-
tical direction with an additional gradient before applying the counterflow. State selective
optical dipole potentials, in which the potential only affects one of the BEC components and
is “invisible” to the second, can provide an alternate way of inducing RT and KH instabilities. For example, an interface between the two components can be created with the use of a light-sheet which is felt by only one of the components. Both RT and KH instabilities can lead to turbulence and in the case of our two-component BEC, quantum turbulence, indicated by the presence of vortices, vortex lines and vortex tangles.

Quantum turbulence is another prominent direction into which our two-component counterflow experiments can be expanded by using a trap geometry with weaker radial confinement so that radial dynamics contributes (as opposed to the effectively one-dimensional dynamics in most of our previous experiments). When the extremely regular MI pattern is allowed to evolve in the absence of external gradients, we often observe features in the condensates which are indicative of vorticity (see fig. 5.6 and fig. 5.9). Since quantum turbulence is theoretically more tangible than classical turbulence, experiments in this direction [14, 109] can prove to be highly productive. theoretical treatment on quantum turbulence in counterflowing BECs, see [101].

The aforementioned state selective dipole potentials can also be used to further study dark-bright soliton interactions with external barriers. Intriguing theoretical results on this front have been made [105]. For example, collisions between a dark-bright soliton and an external attractive potential well present in only one of the components of the soliton produces an effective repulsive potential in the second component, leading to partial trapping and partial reflection in the second component. Experimental confirmation of these theoretical results is planned as a future direction of research.

Further studies involving fermions can also be made using our apparatus. These include shock waves in a Fermi gas at unitarity, a topic which is currently under active research [78] and hydrodynamics in Bose-Fermi mixtures.

It is clear that quantum hydrodynamics is an exciting and rich topic. While aspects of quantum hydrodynamics, such as collective oscillations or vortices, have been at the center of BEC research since the very beginning, much remains to be discovered as we progress towards increasingly complex systems including multicomponent BECs, fermions,
Bose-Fermi mixtures, or even systems with synthetic gauge fields (the topic of a future thesis!).
Appendix A

A.1 Homemade Enriched $^{40}$K Dispensers

This appendix includes a description of our adventures in designing and making enriched $^{40}$K dispensers. The motivation for this is that commercial dispensers for enriched $^{40}$K have become outrageously expensive and some commercial batches did not work well. Thus, following the recipes found in [110] and [111], we made our own.

Potassium atoms are released when potassium chloride reacts with calcium according to the reaction

$$2\text{KCl} + \text{Ca} + \text{heat} \rightarrow 2\text{K} + \text{CaCl}_2.$$ 

The reactants are kept in nichrome housings and the reaction is made to proceed when a current flows through the nichrome. Because of the alloy’s high resistivity, ohmic heating allows the above reaction to take place.

After considering several designs for the dispenser housing, we decided on a design which uses premanufactured nichrome tubes. A long Ni80/Cr20 tube with 3 mm outer diameter and 2.5 mm inner diameter was purchased from Goodfellow and cut into three 5.5 cm long segments. Using a tube geometry eliminates the need for bending and spot-welding, as would be the case with using nichrome foils. The ease with which the housing can be loaded using a small funnel placed into the round opening is an important factor for us since everything must be assembled in awkward positions in a glovebox while wearing clumsy gloves. In addition, enriched $^{40}$K is very expensive so losses should be minimized.
To make the housings, 1.5 cm segments of the upper portion of each dispenser are cut off from either end. The portion which is still attached to the tube is flattened into leads. One end of the tube section is then crimped tightly shut while the other end is left open for loading and releasing K atoms. Initial tests performed on this housing show that the rate at which atoms are released is not constant and a large hysteresis exists when turning down the dispenser current. Also, a very large current of 9 A had to be run through the dispenser before any atoms were released. Thus, a modified housing was made in which a small slit approximately 1 mm wide is cut into the top of the tube section. A sketch of our dispenser design is shown in figure A.1. The leads can then be bent and angled in ways which allows the slit to shine towards a preferred direction when the dispenser is held in its mount. Figure A.2 shows the finished dispenser held in its copper mounts.
To keep contaminants from entering the dispensers and to keep the calcium from reacting with the moisture and oxygen in the air, we prepare the dispensers in a glovebox filled with argon. For testing the procedure, we crush and grind about 10 mg of non-enriched KCl to a fine power using a mortar and pestle. Dendritic calcium\(^1\) must be filed to a fine powder before it can be mixed with the crushed potassium chloride. The filing of the calcium is by far the most laborious part in making the dispensers. Following [111], we file the calcium pieces using a jeweler's file while holding onto them first with a pair of tweezers, then pliers, at times vigorously, over a \#100 mesh into a glass container. The calcium powder is then mixed thoroughly with the crushed KCl in a volume ratio of 2:1 (by eye) and loaded into the housing with the help of a small funnel.

After successfully testing the dispensers with unenriched potassium (from Alfa Aesar 1446, 99.95%) and 99.5% pure calcium shot (from Sigma-Aldrich), we repeated the process using distilled calcium (99.99% pure from Sigma-Aldrich) and 9% enriched ⁴₀K (purchased from Trace Sciences), this time loading the mixture into the dispenser housings shown in figure A.1. We were able to fill two housings with 10 mg of the enriched ⁴₀K. To prevent the mixture from falling out as we handle the dispensers and place them into their mounts, a layer of aluminum foil was wrapped around the body of the dispenser while the open end was plugged with a 4-40 setscrew. Any leftover KCl and calcium are placed in jars filled with argon gas which are in turn, stored in vacuum containers (Sample-Storr) purchased from Ted Pella.

\(^{1}\)for testing we used calcium which was not ultra pure, but for the final dispensers ultra pure calcium was used
Appendix B

B.1 Lithium Heat Pipe Spectroscopy Cell

We describe in this section our experiences in building a lithium heat pipe spectroscopy cell. The design of the cell follows that found in [34] and [112].

The heat pipe is constructed out of stainless steel tubing with $\sim 0.65''$ ID. Approximate dimensions of the finished spectroscopy cell are given in figure B.1. As can be seen in this figure, the ends of the heat pipe are capped with CF-16 windows allowing for optical access. The ends are also at a slight angle (about $5^\circ$) from the long axis of the pipe to prevent unwanted back reflections.

Since lithium corrodes silica based glass [113], the ends of the heat pipe are liquid cooled (with PrimoChill PC ICE) to prevent lithium from migrating towards the windows at either end. Small cylindrical reservoirs welded to the heat pipe approximately 2" from the viewports on either side carry heat away from the cell. These reservoirs are connected to a cooling unit powered by a pump and heat exchanger intended for cooling overclocked CPUs. As an additional safety measure, we fill the cell with about 20 mTorr of argon buffer gas to help prevent lithium from bombarding the viewports. Within the central region of the heat pipe we place a piece of #100 mesh curled into a tube so that it is about 3 layers thick to help guide lithium condensed on the walls back to the center by wicking action.

We purchased $^6\text{Li}$ and $^7\text{Li}$ from Sigma-Aldrich. As lithium is highly reactive with water and oxidizes extremely quickly in air, loading of the lithium into the heat pipe was achieved in...
a glovebox filled with argon. We found that even while working under an argon atmosphere, a thin layer of oxide tended to form on the surface of the $^6$Li chunks. The $^7$Li came in the form of rods from which smaller pieces could easily be cut. The slightly oxidized surface layer can easily be scraped off using a scalpel. Chunks about 1 cm or less in diameter were placed into the heat pipe with the aid of a long spoon. Care was taken to ensure that the lithium chunks would not block the spectroscopy beams. The $^6$Li came in chunks submerged in mineral oil and must be cleansed prior to being placed in the heat pipe. To clean the chunks, we remove one piece at a time from the mineral oil and place the chunk into a hexane bath. The chunk is agitated for several seconds before it is washed again in a second hexane bath. A third hexane bath was used for the final procedure. Then it is blotted dry with KimWipes. We proceed to scrape off any oxide layer which may have formed on the surface with a scalpel. It was helpful to stabilize the lithium chunk using a pair of pliers while scraping. When we are satisfied, the chunk is placed onto the wicking mesh near the center of the heat pipe using a long handled spoon. The entire procedure is performed in an argon-filled glovebox and for safe measure a constant stream of argon was always directed through the pipe as well as onto the Li chunks during the scraping. About 0.2 g of $^6$Li and

**Figure B.1:** Picture of lithium heat pipe spectroscopy cell.
0.2 g of $^7\text{Li}$ was used to make this spectroscopy cell.

Heat tape is wrapped around the central portion of the heat pipe, near the area where the lithium sits. To activate the lithium, the central region of the heat pipe is brought to a temperature of about 340°C and kept at this temperature at all times. Standard saturated absorption spectroscopy like that described in 2.3 is used to locate the absorption lines.

An initial attempt at locating the lithium absorption lines failed miserably due to (after a time consuming search) a leaky glass-to-metal seal in one of the CF-16 windows purchased from MDC. Upon examination, the lithium had completely oxidized and was deathly black. A second attempt after replacing the window produced more promising results. We also discovered that the lithium acted like a getter as the pressure in the cell would drop a few mTorr over the course of minutes. Over the course of weeks, however, the cell pressure would increase to the point where the height of the crossover peak would decrease until it was barely visible, but the Doppler profile would still be present. Decreasing the cell temperature resulted in an overall signal drop while increasing the cell temperature did not produce stronger signals either. Only when the pressure in the heat pipe was decreased by pumping out the cell did we again see a strong crossover peak signal. We suspect that residual outgassing from the Li might be the reason. Unlike the Li used for the source described in Appendix B.2, we have not yet baked out/degassed the Li used for the spectroscopy cell.

**B.2 Lithium Source**

Our lithium source for the Li MOT of our second apparatus is based on the design of [34] and is shown in figure B.2(a). Preparing the Li oven for attaching the machine took several steps.

1. prebake oven at 400°C for $\sim$ 6 hours following an ultrasonic cleaning

2. fill oven with Li under an argon atmosphere in glovebox

   - for $^6\text{Li}$ chunks, rinse mineral oil off with three consecutive hexane baths and scrape
outer oxidized layer off with a scalpel; all done while a stream of Ar flows onto the lithium pieces

- for $^7$Li, scrape off outer oxidized layer

3. seal off oven with blind flange and nickel gasket

4. do second bakeout, this time at 540°C for $\sim$ 1.5 hours with cold trap (made from a single coil submerged in liquid nitrogen) connecting the source to a turbo pump

The oven was filled with as many chunks of lithium as would fit, about 4 g each of the two isotopes. The coldtrap and pump placement during the second bakeout is shown in figure B.2(b). It is absolutely necessary to use a nickel gasket during the second bakeout (and when attaching the oven to the rest of the vacuum chamber). A silver plated copper gasket will not survive the bakeout as lithium attacks copper, a process which is accelerated at higher temperatures.

We also note that nickel gaskets which have been stamped can be less air-tight than gaskets which have been machined. Ni gaskets also require more force to seal. The stamped gaskets often have scratches and striations which may get in the way of producing a air-tight seal. The vacuum chamber uses two Ni gaskets, both of which have been purchased from SEV VACS in Japan. These gaskets are machined with a grove on the gasket face, which SEV VACS claims to help form a better seal.
After the second bakeout, the oven is valved off and detached from the coldtrap/pump so that it can be moved to its final position. In order to attach the oven to the rest of the machine without exposing the Li, which is kept under an Ar atmosphere, to air, we fabricated a glovebag fitted around the attachment point of the Li oven. The glovebag is slightly over-pressured with argon gas before venting the machine, again with argon, and attaching the Li oven.

To make the glovebag, we cut sheets from a plastic drop cloth and welded the pieces together using a small commercial hand-held bag re-sealer, a heatgun, and a soldering iron. The bag re-sealer was battery operated and was not easy to work with. Though it was able to create an air-tight seal, the thin plastic tore easily while using this device. The heatgun was much easier to work with. By holding a seam close to the hot air blowing from the heatgun and moving along the seam, one can easily weld together the plastic pieces. However, it was much too difficult to hold the plastic at a constant distance from the heat gun and move at a constant speed to create a evenly welded seam. Using the heatgun produced a weld with many small leaks. The best way to weld the plastic was to sandwich the plastic between two pieces of parchment paper\(^1\) and use a soldering iron with a chisel or wide flat tip to run along the seam. The soldering iron allows for great control and creates an air-tight seal. In addition, one can trace out any pattern, making it very easy to create the gloves for the glovebag.

\(^{1}\)waxpaper will also work, but tends to stick to the plastic
Appendix C

C.1 Circuit Diagrams

In this section, we include circuit diagrams of some major electrical components of our machine. All designs and drawings were created by Fred Schuetze in the electronics shop.
Figure C.1: RF VCO.
Figure C.2: 200 MHz AOM amplifiers.
Figure C.3: Current regulator for the fully electromagnetic Ioffe-Pritchard trap.
Figure C.4: Current controller for the fully electromagnetic Ioffe-Pritchard trap.
Figure C.5: Temperature regulator for the fully electromagnetic Ioffe-Pritchard trap.
Figure C.6: H-bridge current servo for HIP trap.
Figure C.7: Laser controller.
Figure C.8: Laser temperature controller.
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