Modeling and analyzing the 2014 mumps outbreak in Moscow, Idaho using the SIR model

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TO THE UNIVERSITY HONORS COLLEGE:

As thesis advisor for Taylor Latimore.

I have read this paper and find it satisfactory.

V. [Signature]
Thesis Advisor

3/21/16
Date
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Modeling and analyzing the mumps outbreak in Moscow, Idaho using the SIR model
Taylor Latimore
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Précis

Mumps virus is a contagious disease that causes intense swelling in the parotid and various other glands. The mumps virus is transmitted through both direct and indirect contacts and can result in infertility and hospitalization in severe cases. Moscow, Idaho experienced an outbreak in 2014 that began with a few students at the University of Idaho and was transmitted quickly throughout the campus. To determine the progression of the mumps outbreak in this town, statistics and disease numbers were gathered from the Idaho Department of Public Health, and trends in susceptible, infected, and removed populations were discovered using a population-independent SIR model. Mumps progression with a population-dependent birthrate was also calculated. Analysis revealed that the mumps epidemic in Moscow is expected to be contained and will eventually die out with much of the population becoming removed. When birthrate was included, the epidemic was also determined to die out, but it is expected to remain endemic and persist at low levels within the population.

It is important to understand mumps and how an outbreak such as the one in Moscow, Idaho will progress. Though relatively mild in most cases, mumps can lead to hospitalization and infertility. These severe complications can be detrimental to an individuals’ physical and psychological well being which are essential to protect. Modeling and analyzing the Moscow outbreak will provide valuable information about how this disease will spread in a small town environment and may help other rural communities better prepare against future mumps outbreaks through education and vaccination.
Introduction

Disease has always been prevalent in society. Due to its powerful effects and broad range of people it can infect, disease has become a highly studied topic in the field of biology. With the addition of mathematical models such as the Kermack-McKendrick or SIR model, it has become possible to predict disease advancement in populations over time in order to determine if outbreaks will be contained or not. One disease that has seen a resurgence due to a variety of factors (such as hesitancy to vaccinate) is the mumps virus. The town of Moscow, Idaho was a victim of the mumps resurgences when 14 people were clinically diagnosed with the illness. In this study, the clinical description of mumps, current vaccine, previous research, and statistics from the Moscow outbreak will be discussed. Additionally, the SIR model will be described and utilized to illustrate mumps progression throughout the community. Herd immunity (also referred to as community immunity) will be mentioned as it pertains to the disease.

Background

What is mumps?

Due to the popularity (and often requirement) of the measles, mumps, and rubella (MMR) vaccination, it is common for people to confuse mumps with measles and rubella, or to just simply group all three viruses together. While they do share many characteristics and are caused by members of the same viral family (Rubulavirus) making them great candidates for multi-disease vaccine, these three illnesses do have distinct and individual traits. The mumps virus, *Infectious parotitus*, is a retrovirus that causes intense swelling of the parotid glands of infected individuals. Specifically, the mumps virus is composed of non-segmented, negative-
sense, single-stranded RNA with a host-derived lipid envelope (Brgles et al. 2016). The lipid envelope that is composed mainly of host lipids is an important quality that allows the mumps vaccine (MMR vaccine) to be effective.

The mumps virus is spread between humans either by direct or indirect contact. Direct contact is facilitated through physical contact with infected body fluids such as respiratory secretions or saliva and indirect contact is accomplished through mechanical vectors and fomites (Galazka et al. 1999). Once infected with mumps, the infected individual is contagious for approximately 14 to 18 days (two weeks for the purposes of this analysis). Infected individuals are advised to avoid contact with others for a minimum of five days after salivary gland swelling is first noticed.

Mumps is categorized by non-specific symptoms such as headache, fever, pain, tenderness, and parotitis. Parotitis refers to swelling of one or both of the parotid glands as well as the salivary glands in the cheek and jaw. Initially, this swelling occurs near the lower part of the ear. Because of this initial swelling, mumps can be confused as lymph node swelling leading to incorrect diagnosis. A distinguishing factor of mumps is the incredible swelling of the parotid gland resulting in a jawbone that is difficult to feel and an altered ear angle. The ear becomes pushed outward and upward due to the immense swelling. Though swelling is mostly confined to the parotid glands, swelling can occur in the submandibular and sublingual glands in some instances. In most clinical cases, swelling usually lasts for around five days but can range anywhere from two to ten days.

Once infected, most people recover from mumps without any long-term health issues, however, complications occasionally arise and can lead to hospitalization. Complications
include meningitis (up to 15% of cases), orchitis, oophoritis, mastitis, encephalitis, and deafness (WHO 2014, Maillet et al. 2014). Orchitis refers to testicular swelling that occurs in about 20-30% of infected post pubescent males who usually experience symptoms in only one testis. Though rare, orchitis can cause sterility in infected males. Female specific complications include oophoritis and mastitis. Mastitis refers to swelling of the breasts and oophoritis refers to swelling of the ovaries which also has the potential to cause sterility. Fortunately, both of these complications occur in less than 1% of the infected adolescent and post-pubescent female population. Encephalitis is brain swelling that can be very dangerous, however, it is also quite rare occurring in less than 1% of the population. Although these complications are somewhat unlikely, the consequences are very important as mumps-induced swelling can result in permanent damage such as reproductive infertility.

**Vaccination**

The mumps virus, *Infectious parotitus*, is included in the MMR vaccine to support immunity against measles, mumps, and rubella. Originally licensed to be distributed to individuals in the United States in 1967, this vaccine has been extremely successful in reducing the number of mumps cases countrywide (Anderson & May 1985). According to the Center for Disease Control and Prevention (CDC 2015), from the time of its first administration to present day, the availability of this vaccine has reduced mumps rates in the country by 99%.

Given in one dose (between 12 and 15 months of age), the MMR vaccine is about 78% effective in preventing against mumps. Most medical professionals recommend and many higher education institutions require two doses of this vaccine which is about 88% effective in granting immunity to mumps (CDC 2015). The second dose is usually administered between 4
and 6 years of age, but it can be received earlier or later as long as at least 28 days have passed since the first dose. The CDC (2015) also recommends that persons born on or after 1957 receive at least one dosage of the MMR vaccine if they do not have sufficient evidence of immunity. Adequate evidence of immunity includes laboratory confirmed immunity, documentation of birth before 1957, two doses of the MMR vaccine, or a physician-diagnosed case of the mumps.

International travelers are strongly encouraged to receive both doses of the MMR vaccine before departure, especially, if they are traveling to countries with recently known mumps outbreaks. Some countries that fit this description include Nepal, Japan, China, and Colombia (WHO 2014). Healthcare professionals and others involved in healthcare fields are also encouraged to obtain the full two doses of the vaccine since these workers are exposed at much higher rates than the general population.

**SIR model**

Originally developed by Kermack and McKendrick in the 1930s, the SIR model is a mathematical tool used to model epidemiological changes in a disease of interest. Each population subset is included in the model under one of the letters, S, I, or R. Susceptible individuals are those who are at risk of contracting the disease and are denoted by the letter S. At first signs of outbreak, most individuals in a population reside in this group. As the outbreak progresses, some previously susceptible individuals become infected and can perpetuate the spread of disease. This population subset is denoted by the letter I. Once infected individuals recover and are no longer spreading the disease, they become part of the removed (or recovered) group denoted by the letter R. People who have been vaccinated also constitute a
majority of this group. The three groups S, I, and R, are converted into functions of time and, when summed, comprise the entire population, N. In other words, S(t) + I(t) + R(t) = N where t is the time. Of note, in this model type it is assumed that the population, N, is not changing and is not a function of time. This implies that population factors such as births, deaths, and migration are not considered. The flow of the SIR model is illustrated below.

Rates of individuals entering the susceptible, infected, and removed/recovered populations are very important in the SIR model. The subsequent rate or differential equations are listed below.

\[
\frac{dS}{dt} = -\alpha SI \tag{1}
\]

\[
\frac{dI}{dt} = \alpha SI - \beta I \tag{2}
\]

\[
\frac{dR}{dt} = \beta I \tag{3}
\]

In equation (1), \( \frac{dS}{dt} \) symbolizes the rate that individuals of a population become susceptible. The coefficient, \( \alpha \), refers to the rate of transmission between susceptible and infected individuals. Individuals leave the susceptible population at the density dependent rate \( \alpha SI \) which is reflected in the negative sign in equation (1).
For the second equation (equation (2)), \( \frac{dI}{dt} \) is the rate at which the infected population changes. The additional coefficient, \( \beta \), is related to the period of infection, in other words, the number of days a given person is infectious. In this study, we assume that the infectious period for mumps is 14 days. Since one individual is infected for a period of 14 days, the value of \( \beta \) is \( \frac{1}{14} \) or 0.07143.

In this model, the rate of recovery \( \frac{dR}{dt} \) is dependent on the rate at which individuals leave the infected population since this model assumes people in this category have acquired immunity from the disease. This is reflected in equation (3) above and is dependent on the period of infectiousness and the number of infected individuals. The vaccinated population will be part of the removed population.

Many manipulations can be made to increase the sophistication of the SIR model. Some of these variations include adding population dependent birth and death rates, and secondary infection rates (Brauer 2004). Additionally, the traditional SIR model can be expanded into an SEIR model (susceptible, exposed, infected, and removed) and in the case of potential bioterrorism attacks, a SEQJR model where S is susceptible, E is exposed, Q is quarantined, I is infected, J is isolated, and R is removed population subsets (Brauer 2004). Although additional categories make the modeling more complicated, the SIR model is an effective tool used to analyze numerous situations regarding infectious disease and epidemiology.

Beginning November 19, 2014, a mumps outbreak began to form in Pullman’s neighboring city Moscow, Idaho. Since that date, there have been 13 lab-confirmed cases in Moscow, 6 probable cases in Boise, and 2 cases in Washington according to the Idaho
Department of Health (2015). Since there is a potential for major complications from mumps (such as sterilization of reproductive organs), it is important to understand this disease and attempt to control it. With a population model such as the SIR model, we can roughly predict the proportion of the Moscow population that is susceptible, needs to be vaccinated, will become infected, and will recover and become removed.

Previous research

In recent years, mumps cases have been on the rise in school and university settings as these institutions are breeding grounds for the virus despite vaccination requirements. Contributing factors to this increase in prevalence include the idea that some students are able to bypass MMR vaccination and that the vaccine is only 78% effective in the population groups that only receives one dosage.

One of the largest mumps outbreak in the United States occurred in 2006. Beginning in Iowa in January, the virus quickly spread to 40 states by April resulting in 2786 reported cases spread throughout the country (Dayan et al. 2008). In December of the same year, the total case number had increased to 6584 with 85 hospitalizations and zero deaths. Out of these cases, it is important to note that no significant outbreaks were reported at primary or secondary schools (Dayan et al. 2008) while most outbreaks were initiated and then further propagated at college campuses. This study illustrates how imperative it is to continue studying mumps especially in university settings since these areas tend to have high rates of transmission among the population.

Another more publicized outbreak of mumps occurred in 2014 among many members of the National Hockey League (NHL). The outbreak first made headlines when star Olympian
and professional player Sidney Crosby of the Pittsburgh Penguins was diagnosed. The disease spread to approximately 17 other players (Melnick 2014). Professional hockey players come from and travel all over the world. Some of these destinations do not require their residents to be vaccinated for mumps. In fact, according to Gregory Wallace of the Center for Disease Control and Prevention (CDC), almost half of the world does not even use the MMR vaccine (Reisz 2014). In addition to interaction with potentially unvaccinated individuals, players are in close proximity to one another the majority of their time, and share planes and hotel rooms during travel. These factors helped facilitate the rapid spread of mumps within the NHL (Reisz 2014). It is important to study outbreaks such as this one especially when analyzing university settings and small towns. In areas such as the University of Idaho and the city of Moscow, residents and students alike are often in close contact the majority of their days. Additionally, there is an influx of international students who may or may not have previously received at least one dose of the MMR vaccine. Conditions such as those in the NHL can lead to mumps outbreaks such as the current one being analyzed in this thesis.

One study conducted by Barskey et al. (2012) examined a mumps outbreak that occurred in New York during 2009 and 2010 where most cases involved Orthodox Jewish adolescents who were attending a secondary school together. The close contact for an extended period of time that is characteristic of schools is a likely cause for the quick spread of the virus. During this outbreak, 140 complications arose with the most common being orchitis (found in 7% of patients) (Barskey et al. 2012). Fortunately, no deaths or encephalitis were reported, but nearly 41 people were hospitalized for orchitis or painful parotitis causing immense discomfort (Barskey et al. 2012).
Mumps outbreaks are not exclusively found in the United States. From October 2012 to September 2013, there was an epidemic in Greater Manchester, England, another country that highly promotes vaccination for measles, mumps, and rubella. During this time frame, over 1,000 cases were reported with 395 confirmed in a laboratory, 91 deemed probable, and 312 considered possible (Pegorie et al. 2014). Interestingly, researchers in this study found that two cohorts with the greatest number of mumps cases reported were infants who were too young to receive the vaccine and children aged 10-19 who likely did not receive the vaccine because of misinformation about the MMR vaccine causing autism (Pegorie et al. 2014). Secondary schools saw the greatest number of cases compared to any other setting. Fortunately, this study illustrates that if proper measures are taken such as educating the public, encouraging vaccination in those receiving one or no doses, and examining vaccination records for those in high risk situations, the outbreak can be contained and eventually reach a steady state within the population.

Most recently, on February 19, 2016, two universities in Indiana, Butler University and Indiana University – Purdue University Indianapolis, announced that the mumps virus was being spread among students on their respective campuses. Butler University confirmed that a total of nine students had contracted the virus since February 11 and Indiana University declared a total of four confirmed cases since February 10 (Bagget 2016). As noted before, vaccination is 88% effective in two doses. But, some individuals only receive one dosage resulting in only 78% immunity. The additional 22% of members in this population have a heightened chance of contracting the disease than those with two doses. Proper measures are currently being taken on the campuses listed above in an attempt to prevent transmission. These measures include
quarantine of infected students and easier access to vaccine clinics. With this diligence, the mumps outbreak in Indiana will likely subside over time.

SIR models have been used to effectively evaluate disease progression since their development in the 1930s and have had a major impact in the field of mathematical biology. One study conducted by St Clair (2006) used this model to analyze the Iowa disease outbreaks in 2006 discussed earlier. As the observed numbers later supported, St Clair’s SIR model showed that mumps prevalence would decrease after peaking around 35 weeks. The analysis also revealed that the original susceptible population predicted by the Iowa Department of Public Health was an underestimate since the disease would have followed a significantly different trajectory given the initial conditions provided by the department (St Clair 2006).

*Moscow, Idaho Statistics*

In an article released in the Moscow-Pullman Daily News, author Danielle Wiley (2015) highlighted the issue of mumps spreading throughout the Northwest especially within the University of Idaho student body. This article highlighted three original cases diagnosed in students attending the university. Prompting an investigation, the Idaho Department of Public Health began publishing press releases. Ten lab-confirmed cases in college students and elementary school-aged children were reported in a November 2014 press release (Idaho Department of Public Health 2014). Two additional cases were later reported resulting in 12 total infected individuals in 2014. In addition, one case in a middle-aged man was reported in 2010. A table of all reported laboratory confirmed mumps cases per year can be seen in Appendix A (page 26).
Modeling the Mumps Outbreak

Methods

Data was gathered from the Idaho Department of Public Health’s Immunization Records and Infectious Disease cases (see Appendix A). Additionally, contact was established with Idaho’s District 2 Department of Health to verify mumps case numbers. District 2 includes the Latah, Nez Perce, Lewis, Clearwater, and Idaho counties and Moscow resides in Latah County. With vaccination estimates for the state of Idaho supplied by the Department of Public Health in 2013, vaccination estimates for the city of Moscow were calculated using proportional analysis. Approximately 91.1 people out of 233 sampled in the Idaho population have received one or more doses of the MMR vaccine which translates to about 40% of the population or about 9592 people. The vaccine efficiency percentage for one dose of MMR (78%) was then used to determine how many members of the population are considered removed or unable to acquire mumps. From these calculations, 7482 people in the city of Moscow should not contract mumps and thus can be considered immune.

While 7482 people are categorized as immune to mumps, the remaining population is still susceptible. The groups initially consisted of approximately 17052 people at the beginning of the outbreak in 2010. This susceptible population then decreased from 2009 to 2015 (see Appendix A). To determine the transmission rate ($\alpha$ in the SIR model), a graph of the infected and susceptibles populations was created and the slope ($\frac{g}{\alpha}$) was used. The mathematical analysis used to calculate this slope is located in Appendix B (page 29). Upon defining the coefficients for the respective S, I, and R equations for the SIR model, disease progression was graphed using a student access version of Matlab.
Another aspect of this study was to examine disease progression given steady states that include a birth rate. The birth rate, $\Upsilon$, was acquired from the natality reports of the Idaho Department of Health and Welfare’s website. District 2 had the lowest rates in the state in 2013 with a birth rate of 10.9 per 1000 people. This was converted into a percentage of 1.09% leading to a $\Upsilon$ value of 0.0109. Susceptible and infected equations were modified to include perturbations ($\delta$ and $\epsilon$) and were solved using integration methods and the quadratic equation. Disease progression and stabilization of the system with birth rate included were then analyzed.

**Figures**

![Number of Mumps Cases per Year in Moscow, Idaho](image)

**Figure 1. Number of mumps cases per year in Moscow, Idaho.** Illustrated above are the number of mumps cases reported to the Idaho Department of Public Health from 2010 to 2015. There was a period of no reported cases from 2011 to 2013 prior to the outbreak at the University of Idaho in 2014. The graph was created using Microsoft Excel.
Figure 2. Number of infected individuals as a function of susceptible individuals. The above graph shows the peak number of individuals who contracted mumps from 2010 to 2015 based on the number of susceptibles. The parabolic curve shows that after a peak at 12 infecteds, the number of those infected decreases as the number of susceptibles decreases. The graph was created using Microsoft Excel.

Figure 2 (above) illustrates the relationship between supplied numbers of infected and susceptible individuals. Counts of infected persons were supplied by the Idaho Department of Public Health and susceptible numbers were calculated using the analyses described in the methods section. This figure shows that when the number of susceptible people in Moscow, ID is high, there are few infected individuals. This is the characteristic of the beginning of an epidemic. As more people become infected, the number of susceptibles begins to decrease as a fraction of this group is moving into the infected group. Further, after the infected population has reached its maximum, there are continually fewer susceptibles as they have moved from being susceptible to being characterized as removed or infected. This graph also illustrates that
when there are no longer any persons with confirmed mumps cases, the susceptible population is smaller than its initial value.

**Figure 3. Determining slope from susceptible and infected populations.** The figure above shows the graph used to calculate the transmission rate ($\alpha$) of mumps from infected and susceptible populations and the infectious period (14 days). Using a linear line of best fit, the slope was calculated to be 453.16 resulting in an $\alpha$ value of 0.030894. The correlation coefficient for this line was 0.57594. The graph was created using Microsoft Excel.

To determine the rate of transmission of mumps within people in Moscow, Idaho a linear analysis was utilized. The graph above (Figure 3) shows the linear line of best fit calculated using Microsoft Excel. Although the correlation coefficient was 0.57594, the linear relationship was still strong enough to be considered accurate.
Results

Model Projections

The population independent SIR simulation of the mumps outbreak in Moscow, Idaho is shown below. It is important to recognize that birth, death, and other population factors were not included in this model.

Figure 4. Progression of mumps outbreak. This graph illustrates how the three categories of individuals within the Moscow population (susceptible, infected, recovered/removed) will be affected as time continues. Susceptibles will decrease rapidly and then eventually level out at zero. The infected group is expected to increase as the susceptible population decreases, but then peaks and falls back to zero and more people move to the recovered group.
Illustrated by Figure 4 (page 20) the SIR simulation reveals that the mumps outbreak is expected to die out within the year. The most dynamic section of the graph occurs before the 0.05-year marker indicating that mumps has the greater impact on the population at the very beginning of outbreak. As predicted by the model, the mumps virus does appear to have died out with susceptible and infected populations decreasing substantially and removed populations greatly increasing. No new cases have been reported since the fall of 2014.

*Adding a Birth Rate*

The calculated birthrate, $\gamma$, for Idaho’s District II supplied by the Idaho Department of Health and Welfare was used in this analysis. This value was 0.1526. The birthrate was included in the susceptible rate equation. Calculation and mathematical principles involved in this process can be visualized in Appendix C (page 30).

*Figure 5. Mumps progression with population birth rate included.* In the graph above, the mathematical progression of a mumps outbreak with similar characteristics as the data in this
thesis is illustrated. The Moscow epidemic follows this oscillatory pattern with the disease
dampening at \( \frac{\gamma}{\beta} = 0.1526 \) over time.

The conducted analysis revealed that given a constant birthrate of 10.9 births per 1000 people, the number of mumps cases will exhibit an oscillatory pattern with dampening over time until the disease stabilizes at the steady state \( \frac{\gamma}{\beta} = 0.1526 \). Thus, mumps will remain an endemic disease in the city of Moscow, Idaho, but it will persist only at low rates among the population.

Herd Immunity

A common goal of many healthcare workers, researchers, and epidemiologists alike is to eradicate all infectious diseases. Though a grand task, this goal can be accomplished as evidenced through the removal of disease such as small pox and polio from society (Fine 1993). There are many factors that can help lead to disease eradication. One of the main factors is a principle known as herd immunity. As implied by the name, this principle suggests that members of a population (herd) can receive indirect immunity from a disease given a sufficient amount of individuals within the same population are vaccinated for or have natural immunity from the disease (National Institute of Allergy and Infectious Disease 2010). This is due to the idea that people who would normally be susceptible (unvaccinated) have a significantly reduced chance of acquiring a disease because they are less likely to come in contact with an infected person resulting in fewer opportunities for an outbreak to occur (Fine 1993, NIH 2010).

For each infectious disease, there is a certain critical value that predicts how many population members must be vaccinated for herd immunity to be effective. This critical
threshold has much variation though with values depending on interpretations and calculations
of what constitutes an epidemic (Fine 1993). Additional factors involved in determining herd
immunity values are infectivity, method of transmission, seasonal effects, number of
susceptibles and their demographic information, social interaction habits, and contact rates
estimates (Fox et al. 1971). These elements can help calculate the herd immunity threshold for
a given population.

Percentages falling below the critical threshold required to contain mumps infections
have previously resulted in outbreaks. For example, mumps outbreaks were common at a
United States Air Force Base basic training camp since vaccination levels for mumps among
incoming recruits were lower than what is required to maintain herd immunity (Lewis et al.
2015). The researchers working on this study recommended the Department of Defense
consider vaccinating all incoming individuals susceptible to mumps in order to help decrease
infection and contain any future outbreaks.

Herd immunity is important for an epidemic and should not be taken lightly. Some
members of a population may not get vaccinated for a disease that may become an outbreak
due to a variety of reasons including pregnancy, allergies, compromised immune systems, and
infancy (CDC 2015). For some, it is a choice whether or not to be vaccinated, but for the
proportion of the population noted above, it is important for the critical vaccination threshold
to be reached in order to promote safety.

Conclusion

In the city of Moscow, Idaho, the 2014 mumps outbreak is expected to die out in the
simple SIR model with much of the population becoming removed. If including a birthrate, the
disease remains endemic in the population, but only at a low level that does not pose significant epidemiological threat. These model predictions are consistent with real life results. It is important to understand the mumps progression in this situation since this disease can become prevalent among communities with strong and frequent social interactions such as universities and the towns they reside. Studying and analyzing the course of mumps provides valuable information about trends expected in the community and if a significant portion of the population is vaccinated a complete outbreak can be prevented. Overall, the mumps outbreak in Moscow, Idaho, is controlled and the SIR model was an effective way to predict and model the diseases’ development.
References:


Appendix A

Number of mumps cases, susceptible, and infected per year in Moscow, Idaho

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Table of Vaccine Estimates
NATALITY

U.S. The U.S recorded 3,932,181 live births in 2013, down less than 1 percent from 3,952,841 live births in 2012\(^1\). The 2013 birth rate was 12.4 per 1,000 population, compared with 12.6 per 1,000 population in 2012. The 2013 general fertility rate of 62.5 births per 1,000 women aged 15-44 was a historic low for the United States\(^1\).

Idaho Following decades of consistently higher birth and fertility rates, Idaho's birth rates fell considerably in the 1980’s to converge with the U.S. rate in 1988 for the first time. After three years below the U.S. rate, Idaho’s birth rate rose above the U.S. birth rate again from 1992 through 2013. In 2013, the birth rate for Idaho was 13.9, a 3.5 percent decrease from 14.4 in 2012. The fertility rate decreased 3.1 percent from 74.0 in 2012 to 71.7 per 1,000 women aged 15-44 in 2013. The number of births decreased by 593 births (2.6 percent), from 22,941 births in 2012 to 22,348 births in 2013.

Idaho live birth rates in 2013 ranged from a low of 10.9 per 1,000 population in District 2 to a high of 18.5 per 1,000 in District 7. All 7 districts experienced lower birth rates in 2013 from 2012. District 7 had the largest rate decrease (5.6 percent) from 19.6 to 18.5; District 4 decreased 2.4 percent from 12.5 in 2012 to 12.2 in 2013.

Race and Ethnicity of Mother Beginning in 2004, more than one race and ethnicity may be reported for the mother, and mother’s race data were collapsed into six race categories: white, black, American Indian, Asian and Pacific Islander, other race, or race not stated. Ethnicity data were collapsed into Hispanic, non-Hispanic, or ethnicity not stated. In 2013, 90.0 percent of Idaho mothers were white, 0.9 percent were black, 1.7 percent were American Indian, 2.1 percent were Asian or Pacific Islander and 5.4 percent reported other race. A total of 15.3 percent of mothers were of Hispanic origin and 84.7 percent were non-Hispanic.

Race and Hispanic origin are reported separately on the certificate; mothers of Hispanic origin may be of any race. The method in which race data are categorized changed in 2004 with the use of the new birth certificate, and data in 2013 are not comparable with race data prior to 2004. In Idaho, if mother’s race is reported as other race and “Hispanic” or “Mexican” is specified as the race, then mother’s race remains coded to “other race”. Not all states follow this method of coding mother’s race. In some states, if mother’s race is reported as “Hispanic” or “Mexican”, the race is coded as white. Because not all states report race and ethnicity to the
National Center for Health Statistics in the same manner, Idaho race data are not comparable with national data. See Technical Notes for more information.

Age of Mother

The mean age of all Idaho mothers was 27.5 years in 2013. For the 7,822 first-time mothers with known age, the mean age was 24.6 years in 2013, compared with 26.0 years for first-time mothers in the U.S. in 2013. The ages for first-time mothers ranged from 14 to 58 in Idaho in 2013. The majority of Idaho births, 58.8 percent, were to mothers aged 20 to 29 in 2013 compared with 59.2 percent in 2012. Teen mothers aged less than 20 accounted for 6.4 percent of births in 2013 compared with 6.9 percent of all live births in 2012. A total of 34.8 percent of mothers were aged 30 or older in 2013, compared with 33.9 percent in 2012.
Appendix B

Mathematical approach to calculating $\alpha$ given $\beta$ and slope.

$$\frac{dS}{dt} = -\alpha SI = S'$$

$$\frac{dl}{dt} = \alpha SI - \beta I = I'$$

$$\frac{I'}{S'} = \frac{\alpha SI - \beta I}{-\alpha SI}$$

$$\frac{dl}{ds} = -1 + \left(\frac{\beta}{\alpha}\right) \left(\frac{1}{s}\right)$$

$$\int dl = \int -1 + \left(\frac{\beta}{\alpha}\right) \left(\frac{1}{s}\right) dS$$

$$I(t) = -S(t) + \frac{\beta}{\alpha} \ln(S(t)) + c$$

At the beginning of the outbreak, $S(t) = S_0$ and $l(t) = l_0$.

$$l_0 = -S_0 + \frac{\beta}{\alpha} \ln(S_0) + c$$

$$c = l_0 + S_0 - \frac{\beta}{\alpha} \ln(S_0)$$

$$I(t) = -S(t) + \frac{\beta}{\alpha} \ln(S(t)) + l_0 + S_0 - \frac{\beta}{\alpha} \ln(S_0)$$

$$I(t) - l_0 = (S_0 - S(t)) + \frac{\beta}{\alpha} \ln \left(\frac{S(t)}{S_0}\right)$$

$$\frac{(I(t) - l_0)}{(S_0 - S(t))} = \frac{\beta}{\alpha} \left(\frac{1}{S_0 - S(t)}\right) \ln \left(\frac{S(t)}{S_0}\right) + 1$$

This equation is of the form $y = mx+b$ where $\frac{\beta}{\alpha}$ is the slope. So, $\alpha$ can be calculated by making a graph and finding the slope of the linear line of best fit.
Appendix C

Determining Steady-States Including Birthrate

\[ S = S_0 + \delta(t) \]

\[ l = l_0 + \epsilon(t) \]

\[ \frac{dS}{dt} = \frac{dS_0}{dt} + \frac{d\delta}{dt} = \frac{d\delta}{dt} \]

\[ \frac{dl}{dt} = \frac{dl_0}{dt} + \frac{d\epsilon}{dt} = \frac{d\epsilon}{dt} \]

Determining \( \frac{dS}{dt} \) using substitution.

\[ \frac{dS}{dt} = -\alpha SI + \gamma \]

\[ \frac{dS}{dt} = -\alpha (S_0 + \delta)(l_0 + \epsilon) + \gamma \]

\[ \frac{dS}{dt} = -\beta\epsilon - \left(\frac{\alpha\gamma}{\beta}\right)\delta = \frac{d\delta}{dt} \]

\[ \frac{d\delta}{dt} = -\beta\epsilon - \left(\frac{\alpha\gamma}{\beta}\right)\delta \]

Determining \( \frac{dl}{dt} \) using substitution.

\[ \frac{dl}{dt} = \alpha SI - \beta I \]

\[ \frac{dl}{dt} = \alpha (S_0 + \delta)(l_0 + \epsilon) - \beta(l_0 + \epsilon) = \frac{d\epsilon}{dt} \]

\[ \frac{d\epsilon}{dt} = \left(\frac{\alpha\gamma}{\beta}\right)\delta \]

\[ \frac{d^2\epsilon}{dt^2} = \left(\frac{\alpha\gamma}{\beta}\right)\frac{d\delta}{dt} \]
\[
\frac{d^2 \varepsilon}{dt^2} = \frac{\alpha \gamma}{\delta} \left[ -\beta \varepsilon - \left( \frac{\alpha \gamma}{\delta} \right) \delta \right]
\]
\[
\frac{d^2 \varepsilon}{dt^2} = (-\infty \gamma) \varepsilon - \left( \frac{\alpha \gamma}{\delta} \right) \frac{d \varepsilon}{dt}
\]
\[
0 = \frac{d^2 \varepsilon}{dt^2} + \left( \frac{\alpha \gamma}{\delta} \right) \frac{d \varepsilon}{dt} + (\infty \gamma) \varepsilon
\]
\[
0 = \lambda^2 + \left( \frac{\alpha \gamma}{\delta} \right) \lambda + (\infty \gamma)
\]
\[
\lambda_1, \lambda_2 = \frac{-\frac{\alpha \gamma}{\delta} \pm \sqrt{\left( \frac{\alpha \gamma}{\delta} \right)^2 - 4(\infty \gamma)}}{2}
\]
\[
\frac{\alpha^2 \gamma^2}{\delta^2} < 4 \alpha \gamma
\]
\[
\Rightarrow \alpha^2 \gamma^2 < 4 \alpha \gamma \delta^2
\]
\[
\Rightarrow \alpha \gamma < 4 \delta^2
\]

This implies that the roots of the equation, \( \lambda_1, \lambda_2 \), are complex conjugates with a real negative part. Referring to the original infected equation, we can now solve for \( I(t) \).

\[
I = I_0 + \varepsilon(t)
\]
\[
I_0 = \frac{\gamma}{\delta}
\]
\[
I(t) = \frac{\gamma}{\delta} + \left[ c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t} \right]
\]

As \( t \) approaches infinity, \( [c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t}] \) oscillates toward 0. Thus, the equation becomes

\[
I(t) = \frac{\gamma}{\delta} \text{ as } t \text{ goes to infinity.}
\]

\( \gamma = 0.0109 \)
\[ \beta = \frac{1}{14} \]

\[ I(t) = \frac{0.0109}{\frac{1}{14}} = 0.1526 \]

Given these calculations, the equation has oscillatory motion that dampens around \( \frac{\dot{y}}{\beta} = 0.1526 \).
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